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Design Study Report
and
Final Report

Contract NAS 5-21170

Development and Fabrication of a
Chargeable Magnet System for
Spacecraft Control

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INTRODUCTION

The work accomplished under the subject contract includes the following:

1. Chargeable permanent magnet technology development

2. Magnetometer development (Mod #4)

3. Horizon scanner breadboard (Mod #9)

4. Implementation of a Magnetic Acquisition System (Mod #11)

5. Development of electronics for dynamic closed loop operation of chargeable permanent magnets (Mod #12)

This report constitutes the final report on this contract and includes the Design Study Report on chargeable permanent magnets as well as summaries of all other work done under the contract.

I   DESIGN STUDY REPORT

VARIABLE PERMANENT MAGNETS

I DESIGN STUDY REPORT - VARIABLE PERMANENT MAGNETS

1.0 Introduction

This design study report describes the work done under
the subject contract on the design of variable permanent
magnets for use in magnetic balancing and control of earth
orbiting spacecraft. These magnets can be used instead
of air core coils or electromagnets in applications where
the objective is to produce, or eliminate, torque on the
spacecraft through interaction with the earth's magnetic
field. Such magnets are particularly useful for on-orbit
magnetic balancing to minimize the effects of magnetic
disturbance torques. Once the appropriate moment is
achieved (by command), the magnet controller is turned
off and the moment will be maintained indefinitely with
no power.

For magnetic control applications, these magnets can
have substantial power advantages where larger magnets
are involved and where frequency of switching is low.

This report deals with design of the magnet for minimum
size and weight, and describes the sensing configuration
used for monitoring the state of the magnet.

2.0 FUNDAMENTALS

2.1 Magnetic Poles

In treating permanent magnets, it becomes convenient to deal with free magnetic poles in analogy to the free charges in electrostatics. A unit pole is defined so that two unit poles with a separation of one centimeter in vacuum would experience a force of one dyne, and the magnetic field strength (H) is defined as the force on a unit pole in this field. The fundamental relations giving the force (F) and the magnetic field strength (H) are

(2-1)
$$F = m H$$

(2-2)
$$H = \frac{m}{r^2}$$

where m is the pole strength and r is the distance from the pole. When two magnets are brought closely together in an end to end orientation, the force between the magnets is given by

(2-3)
$$F = \frac{m_1 m_2}{r^2}$$

provided the separation is small compared to the magnet length.

The effects of magnetic material are measured by the intensity of magnetization I -- which is the number of unit poles per cross sectional area. And because the dipole moment is the pole strength times separation, the intensity is also the magnetic moment density.

(2-4)
$$M = m\ell$$

(2-5)
$$I = \frac{M}{volume}$$

Early experimenters conceived of magnetism as a flow of magnetic flux -- perhaps because of the patterns formed in iron filings by the magnetic field. This flow was characterized by lines of induction which have the same direction as the magnetic field and indicate the flow by their concentration. The induction is the sum of the effect due to H and to I and is defined by

(2-6) $B = H + 4\pi I$

where the factor 4π occurs because a unit pole gives a unit magnetic field at unit distance, and the surface of a unit sphere is 4π. The effects of material are often expressed for linear materials in the more familiar expression

(2-7) $B = \mu H$

2.2 Self-Demagnetization and Geometry Effects

Within a magnetic material, we find an interesting effect which is quite important in our study of permanent magnets. In figure 2-1, a unit magnetic dipole is shown with its lines of induction. As is quite obvious, the direction of the magnetic field off the end of this dipole is aligned with the dipole so that another dipole in this position (shown dotted) would find an aiding field. Now consider a position to the side of this dipole; the dipole field is counter to the sense of the dipole, and a dipole of the same sense would find an opposing field in this region. The implications are quite clearly that: first, magnetic material generates its own demagnetizing field; and second, this effect can be minimized by selecting a long slender geometry.

The same result may be obtained by considering a current loop as the source of magnetic field because the field due to a current loop with infinitismal dimensions is the same as the field due to a dipole of infinitesimal dimensions. The differential magnetizing force due to a differential current element (i $\vec{d\ell}$) at a distance r is

(2-8) $$\vec{dH} = \frac{i}{10} \; \frac{\vec{d\ell} \times \hat{r}}{r^2}$$

where $\hat{r}$ is a unit vector directed from the current element to the point of observation.

The magnetic moment of a current loop may be calculated from

(2-9) $$M = \frac{i}{10} \; (\text{Area})$$

2-2

It should be realized that the magnetic moment of a
current loop is defined such that a current loop with
unit magnetic moment will give unit torque in a field
with flux density (B), but a unit dipole is defined to
give a unit torque with unit magnet field strength (H).

Although these two types of moment are dimensionally
different, they will give the same numerical results
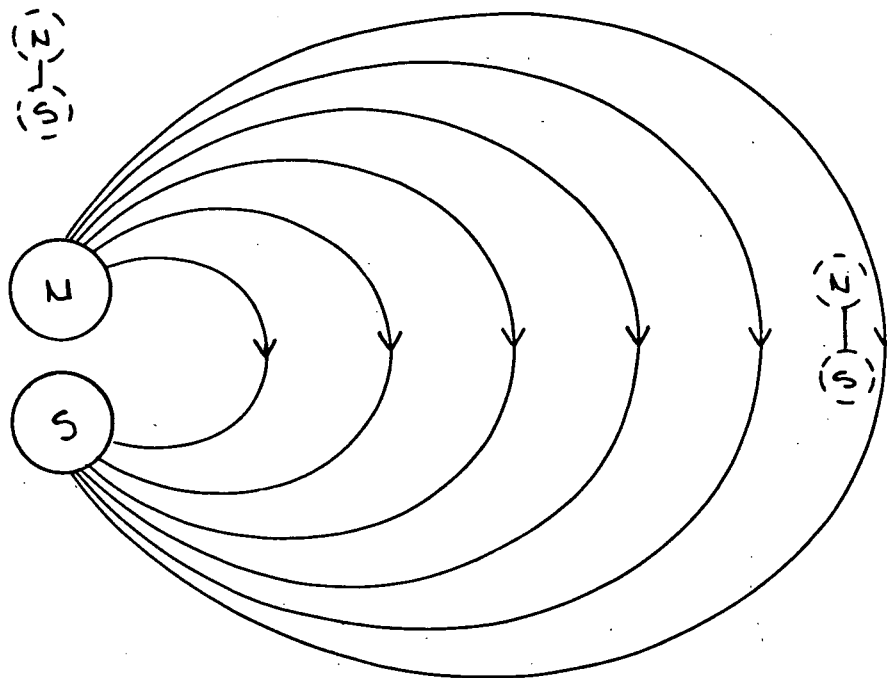in air in systems of units where μ equals one.

Figure 2-1   Field due to a Magnetic Dipole

From this we can readily see that the conversion to mks units is one pole cm equals 10^{-3} ampere meter^2.

Several researchers have investigated self-demagnetization effects, but most magnetic materials are highly nonlinear and quantitative information is usually obtained from graphed experimental data. This data has been summarized to reasonable accuracy by assuming that the self-demagnetizing field near the center of the magnet is proportional to induction and formulating an empirical equation for the proportionality constant. The proportionality constant is taken to be a function of only the geometry and is given for long slender bars[1] by

(2-10)

$$\left(\frac{B}{H}\right)_D = \sqrt{\frac{.49\pi}{2}} \; \sqrt{\frac{p}{a}} \; \left(\frac{\ell}{D}\right)\sqrt{\frac{\ell}{D} + 2\frac{a}{p}}$$

where is the magnet length, D is the width of the bar, pD is the perimeter of the cross section, and aD^2 is the cross sectional area.

Again, this is an empirical relation giving only the self-demagnetizing force near the center of the magnet and makes no attempt to include the effect of properties of the material on the field distribution, but results obtained from this relation have been quite accurate.

Using the assumption that the self-demagnetizing force is proportional to induction in the center of a magnet, we may determine the induction and effective net magnetizing force quite simply by using the load line technique. This technique is best described by example.

[1]Parker & Studders (Ref. 8) page 165
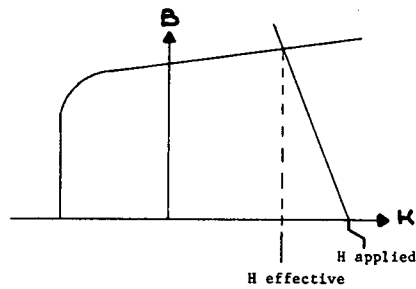
Figure 2-2   Load Line Example

Turning to Figure 2-2, the procedure is simply to draw a load line of slope $-(B/H)_D$ through the H axis at the applied magnetizing force on a graph of B vs H for the material. This line represents the locus of all points which satisfy

$$B = \left(\frac{B}{H}\right)_D (H_{applied} - H)$$

so the intersection of this line with the B-H curve of the material indicates both the net magnetizing force seen by the material, and the corresponding induction.
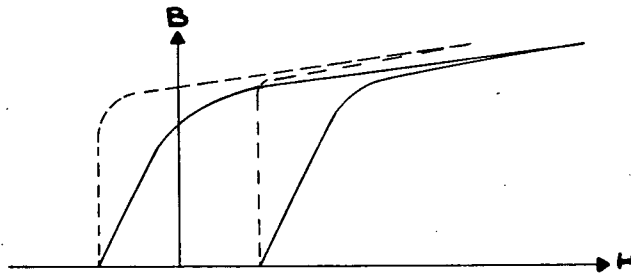
Figure 2-3   Typical Sheared B-H Curve

The same result can also be obtained by redrawing the B-H curve of the material for a specific geometry so that the self-demagnetizing effects are included in the sheared curve. Figure 2-3 shows the effect of shearing on a typical curve shape.

2-5

Before leaving the subject of self-demagnetization, one observation will be noted. This is the effect of shearing on the slope of the curve. It is easily shown either through differential calculus or plane geometry that slopes on the sheared curves can be determined from the original slope, and the load line slope, from

(2-11)

$$\left(\frac{1}{\frac{\partial B}{\partial H}}\right)_{sheared} = \left(\frac{1}{\frac{\partial B}{\partial H}}\right)_{old} + \left(\frac{1}{\frac{B}{H}}\right)_{D}$$

which is to say that slopes add like resistances in parallel. Very small slopes will be unchanged in the sheared curve, but slopes much greater than $(B/H)_D$ will appear in the sheared curve with a slope of $(B/H)_D$.

These considerations will allow us to determine the flux density at the center of the bar, but it remains to determine the effect of the spatial distribution of magnetic intensity (or dipole moment density). As our interest here is with macroscopic effects, we need only a gross measure as an average.

Recalling equation 2-6, if we take averages over the entire magnet we have

(2-12) $Avg(B) = Avg(H) + Avg(4\pi I)$ (from 2-6)

because averaging (arithmetic mean) is a linear process. Fortunately, experimental work has been done to determine the leakage of induction lines from the sides of a bar magnet. From this we can readily determine the average flux density, then using the load line slope we can determine the self-demagnetizing force and if we know the average applied magnetizing force we have the average magnetic intensity.

Figure 2-4 is a graph showing the empirical relationship between average induction in a bar magnet and slenderness ratio ℓ/D.

The significant point is that for ℓ/D ratios greater than 4, the induction averaged over the volume of the bar is 70% of the maximum induction at any point in the bar. The discrepancy between predicted magnetic moment and measured moment for magnets with ℓ/D ratios from 12 to 50 ranged from -1% to +7%.
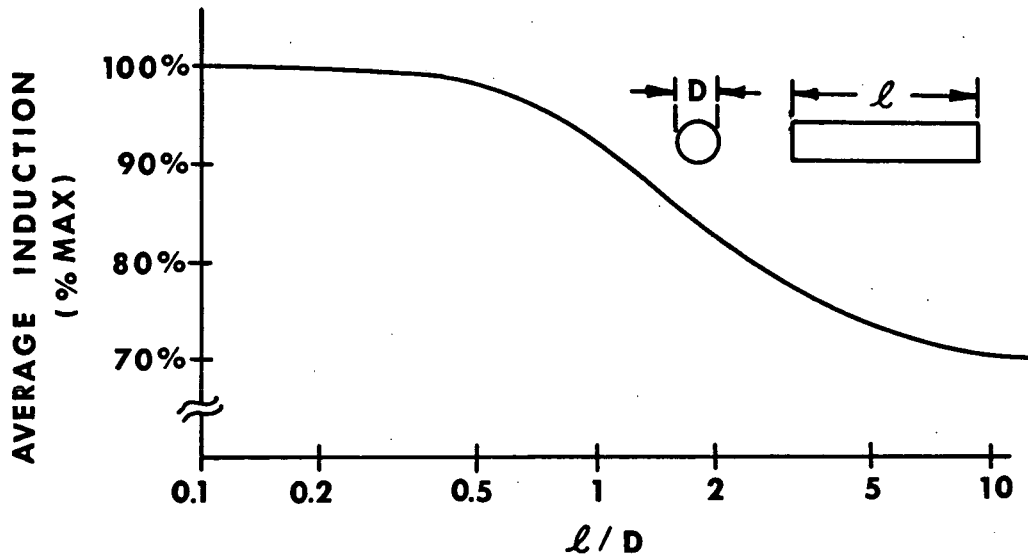
Figure 2-4 Effect of Slenderness on
Average Induction

2.3 Magnetizing Coils

The standard approximation for the magnetizing force due to a coil wound on a long bar is

(2-13)
$$H = \frac{4\pi}{10} \frac{ni}{\ell_c}$$

where ni is the number of ampere-turns, and ℓ_c is the coil length.

In order to determine limits on the validity of this
approximation, we return to equation 2-8 and calculate
the magnetic field on the axis of symmetry for several
current distributions with cylindrical symmetry. The
coordinate system and applicable dimensions are shown
in figure 2-5. Note that a uniform radial current
distribution is assumed for all cases, and that dis-
tance is measured from the coil center so the coil
ends are at $z = \pm \dfrac{\ell_c}{2}$
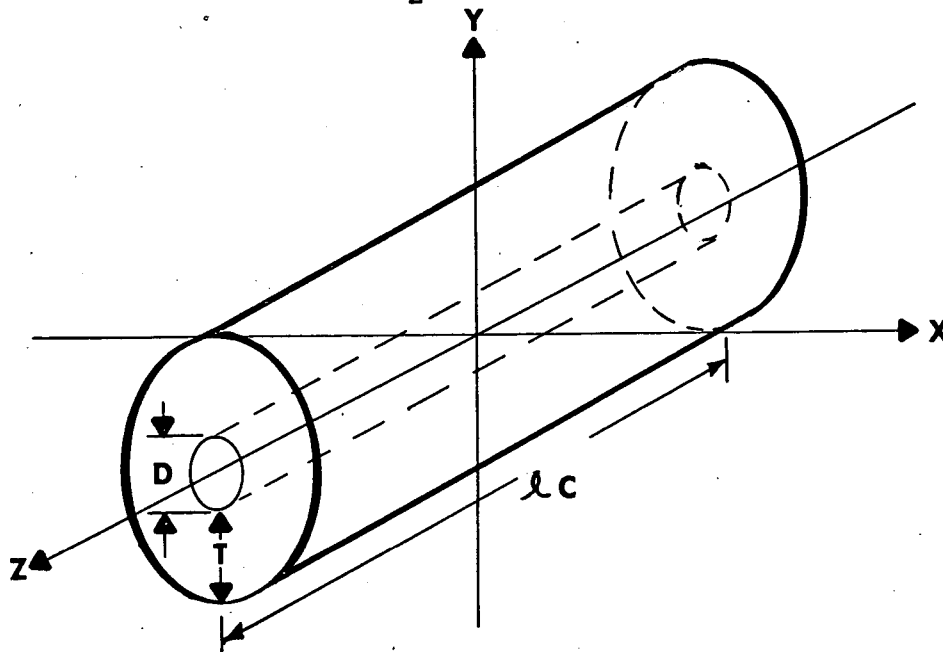
Figure 2-5   Coil Dimensions

Without showing details of the mathematics, the
results are:

for a circular current filament

(2-14) $$H = \frac{4\pi}{10} \ \frac{ni}{D} \ \left[1 + \left(\frac{2z}{D} \right)^2 \right]^{-3/2}$$

for a current disc ($\ell = 0$)

(2-15)
$$H = \frac{4\pi}{10} \frac{ni}{2T} \left\{ \frac{1}{\sqrt{1+\left(\frac{2z}{D+2T}\right)^2}} - \frac{1}{\sqrt{1+\left(\frac{2z}{D}\right)^2}} + \ln\left[\frac{D}{D+2T} \frac{1+\sqrt{1+\left(\frac{2z}{D}\right)^2}}{1+\sqrt{1+\left(\frac{2z}{D+2T}\right)^2}}\right] \right\}$$

for a cylindrical current sheath ($T = 0$)

(2-16)
$$H = \frac{2\pi}{10} \frac{ni}{\ell} \left\{ \frac{\ell + 2z}{\sqrt{D^2+(2z+\ell)^2}} + \frac{\ell - 2z}{\sqrt{D^2+(2z-\ell)^2}} \right\}$$

for a hollow cylinder of current

(2-17)
$$H = \frac{2\pi}{10} \frac{ni}{\ell} \left\{ \frac{2z+\ell}{2T} \ln\left[\frac{D+2T}{D} \frac{1+\sqrt{1+\left(\frac{2z+\ell}{D+2T}\right)^2}}{1+\sqrt{1+\left(\frac{2z+\ell}{D}\right)^2}}\right] \right.$$
$$\left. - \frac{2z-\ell}{2T} \ln\left[\frac{D+2T}{D} \frac{1+\sqrt{1+\left(\frac{2z-\ell}{D+2T}\right)^2}}{1+\sqrt{1+\left(\frac{2z-\ell}{D}\right)^2}}\right] \right\}$$

The cylindrical current sheath has a particularly simple expression for magnetizing force which is convenient to establish bounds on the H distribution of the hollow cylinder. The field distribution of the hollow cylinder will be between the two distributions given by cylindrical sheathes with diameters D and D+2T.

Inspection of equation 2-16 reveals two useful facts. First, the field maximum occurs at the coil center (z=0) and approaches $0.4\pi ni/\ell$ asymptotically from below with increasing length. The length to diameter ratio which gives 10% error is easily shown to be (ℓ/D)=2 while an (ℓ/D) of 7 gives 1% error. This confirms equation 2-13 even for quite short coils.

The field at the coil end is very nearly half the
field at the coil center for (ℓ/D) as low as 2 and mono-
tonically approaches half the maximum field as
(ℓ/D) increases. The variation for intermediate positions
is shown in figure 2-6.
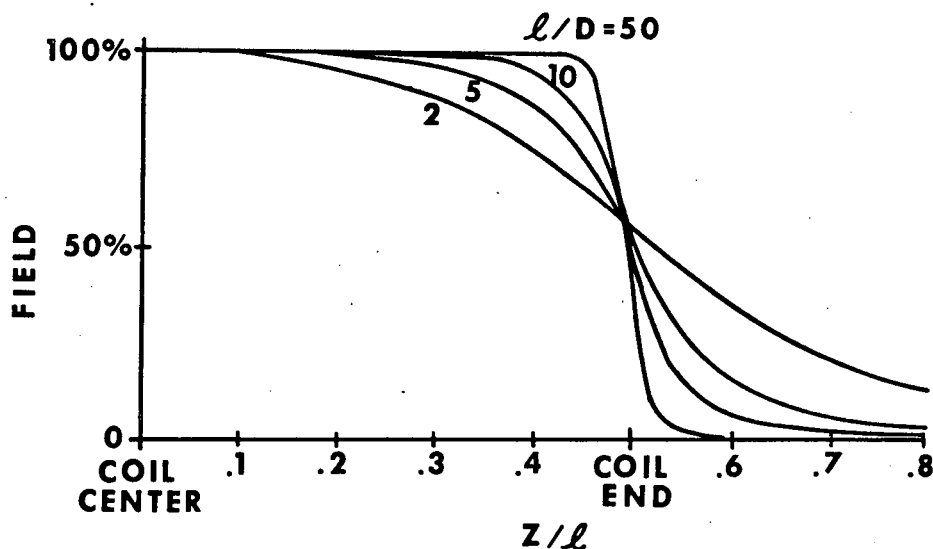
Figure 2-6   Magnetic Field Strength Normalized
            to $0.4\pi ni/\ell_c$ vs position

While equation 2-17 could be graphed in a similar
manner, the addition of a third variable (coil
thickness) provides little additional information.

2.4   Magnetic Moment Measurement

Laboratory measurement of magnetic dipole moment may
be, in principle, accomplished by either making a
torque measurement in a calibrated magnetic field, or
by making magnetic measurements on the magnet.

The most obvious method is a direct torque measurement,
but the torque requirements for small satellites have
historically been in the vicinity of 10^{-2} Newton-meters
(10^{-3} in-lb). It would seem that such delicate torque
measurements could be circumvented by scaling either
the magnet dimensions or the ambient magnetic field,
but both of these approaches have serious drawbacks.

2-10

Because the magnetic core material must be cold worked
to develop the magnet properties, scaling the core
dimensions will require the magnetic material manu-
facturer to scale the size of the cast billet. If
this is not done, the material must be worked to a
different degree and the finished stock will exhibit
different magnetic properties.

The second approach is even less desirable if a sig-
nificant scaling is required. Should the ambient
magnetic field be increased, the magnetic state of
the core material will be altered. As the core
material is highly nonlinear, test results could not
be linearly scaled.

The magnetic moment of a bar magnet can in theory be
determined by averaging the flux density within the
magnet -- but this requires detailed knowledge of the
field distribution within the magnet.

The flux density within a permanent magnet is not
readily measured but another method is available for
determining the average magnetic intensity.

Consider a magnet of length ℓ and the magnetic field
due to this magnet being measured at a distance R as
shown in figure 2-7.

MAGNET   **PROBE**

$$\ell/2$$

$$R$$

$$\ell/2$$
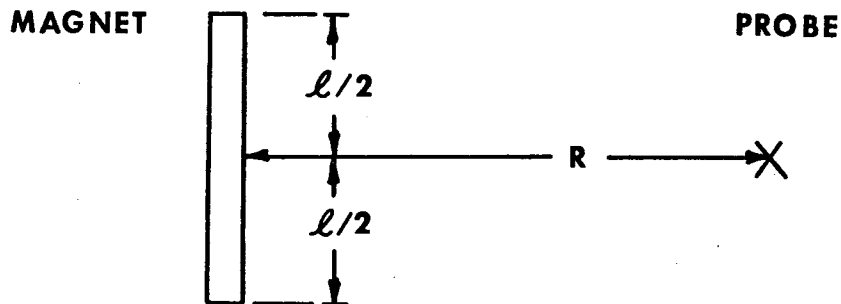
Figure 2-7   Position of Magnet & Test Probe
             for Moment Measurement

If we consider the magnet to be two equal and opposite
poles of strength m and separation ℓ, then the field
at the measurement point will be

2-11

(2-18)
$$H = \frac{m\ell}{(R^2 + \frac{\ell^2}{4})^{3/2}}$$

but B = H in air, and the moment is mℓ. Therefore, we can calculate the dipole moment by making a remote measurement of the magnetic field and calculating the moment from

(2-19)
$$M = BR^3 \left(1 + \frac{\ell^2}{4R^2}\right)^{3/2}$$

This relation will be in error for any magnet because the effective magnetic length is not known precisely, and when the measurement is made very close to the magnet the effect of local flux density variations within the magnet may give significant error. Both of these effects diminish rapidly with increased measurement distance, with the effect of local variations decreasing at least as the fifth power of separation.

Ignoring flux density variations within the magnet for the moment, we may determine the sensitivity of the measurement to probe placement and to uncertainty in the effective length of the magnet. This information can then be used to balance the error caused by the magnetic environment against the errors due to probe placement and uncertainty in magnet length.

Consider two attempts at measuring the dipole moment of a magnet. Let one measurement be error free and the other measurement made with an error δ in probe distance and a relative error γ in determining the magnet length. The relative measurement error is then

(2-20)
$$\varepsilon = \left(\frac{R^2 + \frac{\ell^2}{4}}{(R+\delta)^2 + \frac{(1+\gamma)^2 \ell^2}{4}}\right)^{3/2} - 1$$

2-12

This may be rewritten to show the position error δ which will give an error ε.

(2-21)
$$\delta = \sqrt{\left(\frac{1}{1+\epsilon}\right)^{2/3} R^2 + \frac{\ell^2}{4}\left(\frac{1}{1+\epsilon}\right)^{2/3} - (1+\gamma)^2} - R$$

Figure 2-8 is a graph of equation 2-21 with zero uncertainty in magnet length. This shows the permissible error in probe placement which will cause a +5% error in dipole moment when the magnet length is known precisely. The significant feature in figure 2-8 is the variation of position tolerance with separation. If the probe is placed close to the magnet to obtain a large field, the tolerance on probe position virtually disappears.
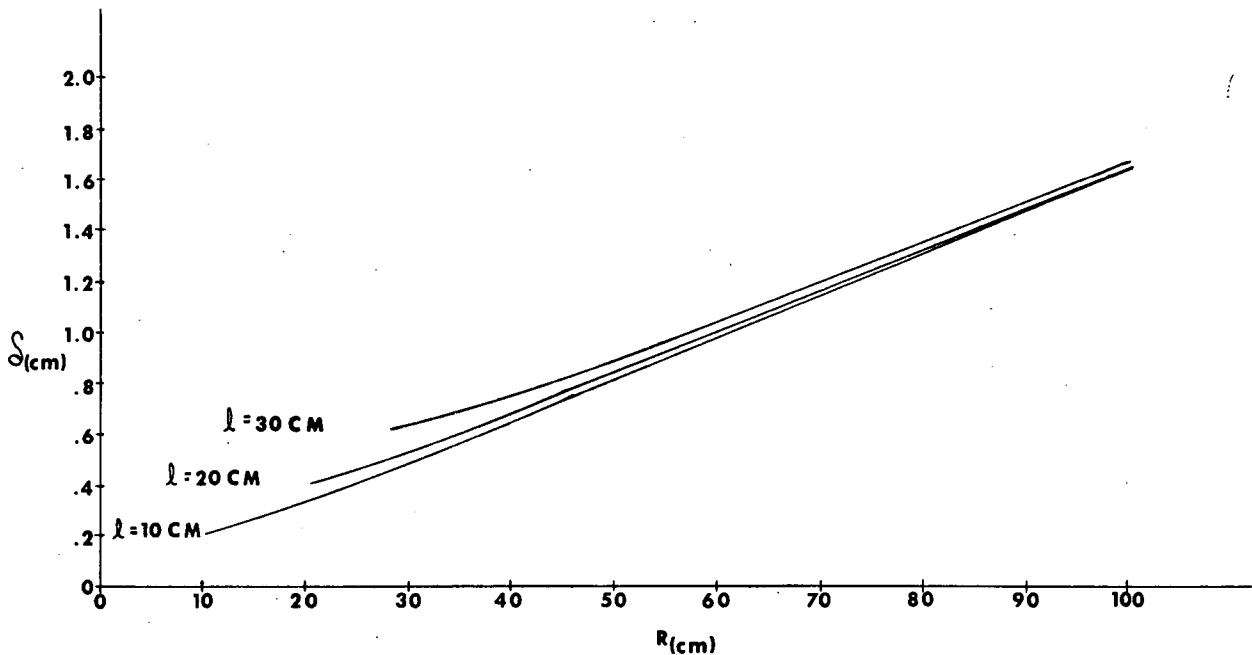
Figure 2-8 Probe Position Error to Give
5% Error in Moment Determination

Figure 2-9 shows the tolerance on probe position to give +5% error in dipole moment when the effective magnet length has been estimated 10% too long. This amount of error in magnet length will give an error of 5% in moment for separation of 1.16 magnet lengths, so the position must be exact to obtain 5% accuracy at this separation and it is no longer possible to obtain 5% accuracy for measurement distance less than 1.16 magnet lengths.
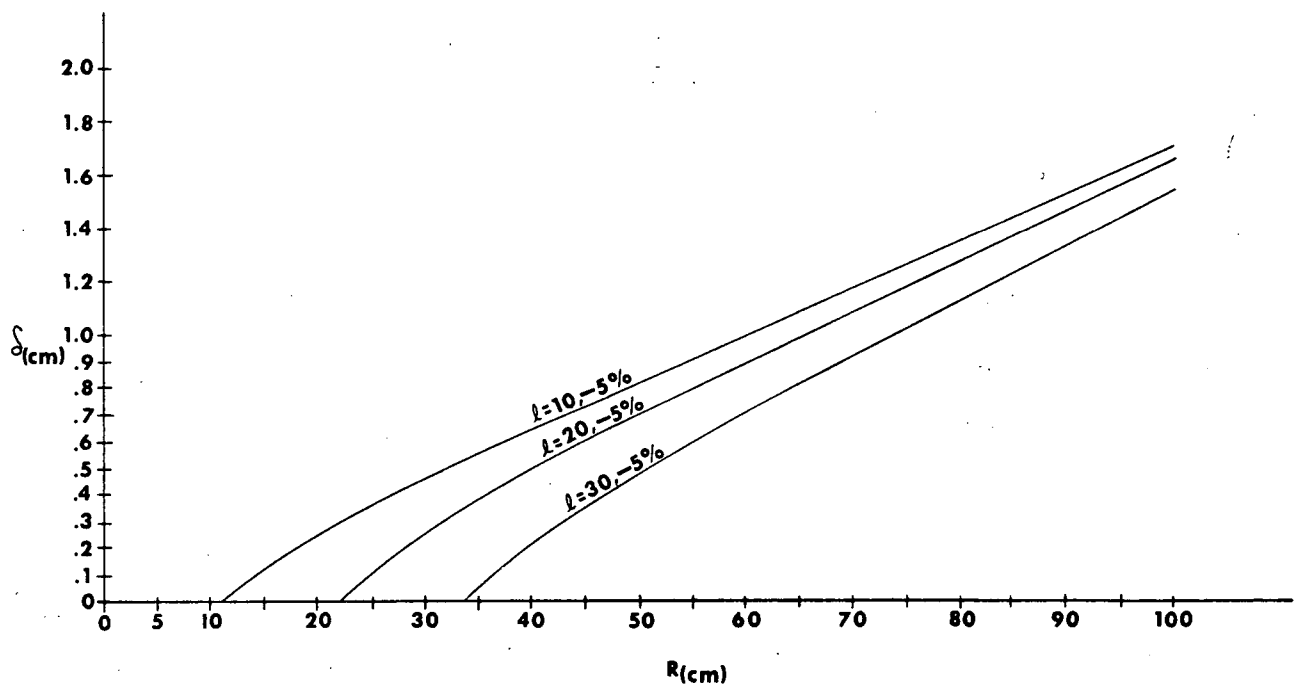
Figure 2-9 Probe Position Error to Give 5%
Error in Moment Determination
Magnet Length is 10% in error.

2.5   Quadrupole Effects

In this section we estimate the effect of non-uniform flux distribution on the field strength due to a bar magnet.

Let us approximate a magnet with the array of equal charges as shown in Figure 2-10.
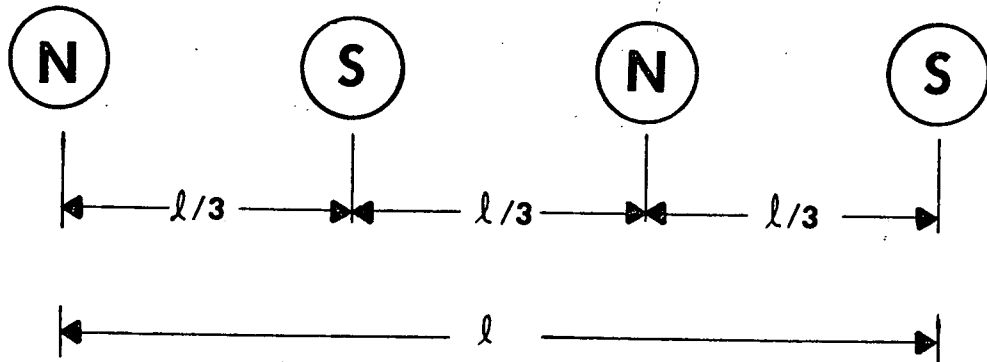
Figure 2-10   Point Charge Approximation to
Bar Magnet

This arrangement of charges may be considered as two dipoles superimposed - the dipole formed by the outer pair of charges form a dipole of moment $m\ell$, and the inner pair of charges form a dipole of opposite sense and moment $\frac{m\ell}{3}$. The net moment is then 2/3 $m\ell$.

The magnetic field due to this arrangement of charges is simply the sum of the contributions by the inner dipole and the outer dipole. A measurement made as in figure 2-7 will indicate a field given by

(2-22)
$$H = \frac{m\ell}{(R^2+\frac{\ell^2}{4})^{3/2}} - \frac{m\ell/3}{(R^2+\frac{\ell^2}{36})^{3/2}}$$

The relative error in determining the magnetic moment from 2-19 is then

$$(2\text{-}23) \qquad \varepsilon = \frac{1}{2}\left[1 - \left(\frac{\dfrac{R^2}{\ell^2} + \dfrac{1}{4}}{\dfrac{R^2}{\ell^2} + \dfrac{1}{36}}\right)^{3/2}\right]$$

which is a function of distance relative to the magnet length rather than a function of absolute distance.

Returning to equations 2-22, notice that the variation of error with distance will disappear if the inner and outer dipoles have the same length. Quite obviously the worst variation with distance occurs with a very short inner dipole. Taking an infinitismal dipole -- again with half the moment of the outer dipole -- the error in calculating the dipole moment from equation 2-19 will be given by

$$(2\text{-}24) \qquad \varepsilon = \frac{1}{2}\left[1 - \left(1 + \frac{\ell^2}{4R^2}\right)^{3/2}\right]$$

which is also a function of relative distance.

Figure 2-11 shows the variation of error with distance for these two cases. Both of these assume an inner dipole with moment equal to half the net moment. The case of an infinitesimal dipole is the limiting case for the relative dipole strengths considered. For either case, the error is less than 5% for measurement distance greater than two magnet lengths.

In view of the uncertainty in effective magnet length, the selected measurement distance should be as great as allowed by equipment sensitivity and by interference from the magnetic environment.

2.6 Characteristics of Magnetic Materials

The origin of magnetic effects in materials lies in atomic dipoles formed by electronic circulation and spin. Classical forces tend to align the atomic dipoles in opposition to an applied magnetic field, but quantum mechanical exchange forces tend to cause the dipoles to align in an aiding orientation. The net result is that the quantum mechanical forces dominate for small groups of atoms which will be aligned aiding, but classical forces will limit the size of these domains to about 10^{-6} cm^3 to 10^{-2} cm^3. Macroscopic effects then, are due to the actions of these microscopic domains.
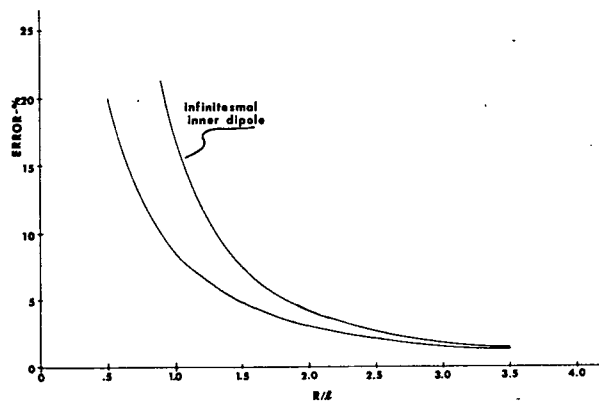
Figure 2-11 Error in Determining Dipole Moment
Due to Quadrupole Effects
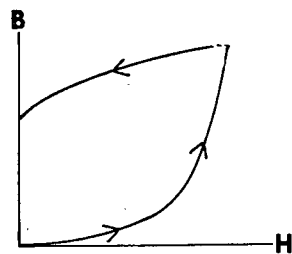
FIG. 2-12A
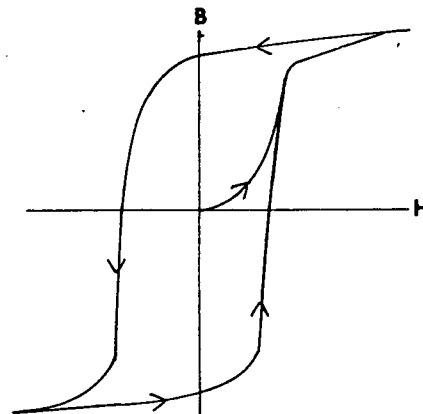
FIG. 2-12B
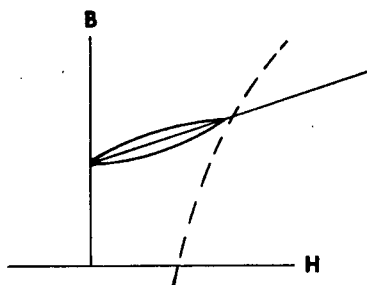
FIG. 2-12 C
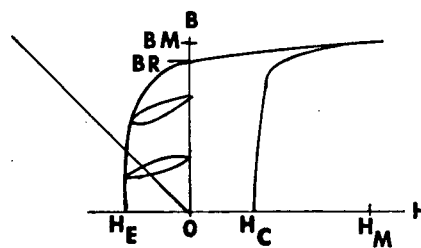
FIG. 2-12D

Figure 2-12 Typical B-H Trajectories

The two outstanding features that one notices in
magnetic materials are explainable in terms of mag-
netic domains. First is the saturation of the mat-
erial which occurs when all the atomic dipoles are
forced into alignment with the field. Because
magnetic effects are due to the orientation of atomic
dipoles, the intensity will not increase past the
point where the magnetizing force causes complete
alignment of all the atomic dipoles.

The other outstanding feature is the manner in which
magnetization changes with applied field. When a
magnetizing field is first applied to magnetic mat-
erial, the domains deform elastically. Then as the
field is increased, the domains in a favorable orien-
tation begin to grow at the expense of less favorably
oriented domains, and this process continues until
either the magnetizing field no longer increases or
until the material is saturated. This action of
elastic deformation alternating with domain growth
gives rise to a multiple valued curve of magnetic
intensity vs magnetizing field as the field is cycled
between some minimum and maximum values.

When a field is applied to a specimen the magnetization
will begin to increase which will cause the magnetiza-
tion to increase more rapidly (Figure 2-12A). As the
applied field is then reduced, not all of the domains
will return to their original (random) orientations
giving rise to residual magnetization. This effect
is sketched in Figures 2-12A and 2-12B which shows the
characteristic shape of the hysteresis loop.

Figure 2-12C shows a minor hysteresis loop which occurs
when the applied magnetizing field is cycled between
zero and a value which does not result in domain growth.
The average slope of this loop is very constant even for
quite large values of magnetization. This average slope
on a graph of induction vs applied field (see equation
6) is termed recoil permeability with symbol μr. The
recoil permeability can be assumed to decrease about 20%
with peak magnetization of 90% of saturation.

Figure 2-12D shows a typical B-H curve shape with
several recoil loops. Notice the terminology: B_m -
maximum induction; H_m - magnetizing force; B_r -
residual induction; H_c - coercive force.

From the preceding, we can identify several desirable characteristics for permanent magnets: the B-H curve of the material should intersect the load line at as high an induction as possible in order to minimize the mass of the magnet; the maximum required magnetizing force should be low to minimize coil requirements; a long slender magnet should be used to give as great a load line slope as possible; and the coercive force should be sufficiently high to minimize external demagnetizing effects.

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3.0 THE MAGNETIC CORE

3.1 Materials

The primary criteria for selecting the magnetic core material are those characteristics that result in a minimum weight system. Materials suited to use in variable magnetic moment applications are alloys with density very near 8 grams/cm^3 -- and a 10% variation is unusual. Thus material density is not a significant factor.

With a choice of materials which have essentially the same mass density, the core material which gives minimum weight is the one with greatest residual induction at the desired operating point. This can be shown by combining equations 2-5 and 2-6 to give

$$(3-25) \qquad \text{volume} = 4\pi \, \frac{M}{B-H}$$

which indicates that the minimum volume magnetic core is one with maximum induction.

The residual induction at the operating point is determined by both the B-H characteristics of the material and by self demagnetization due to core geometry (see next section). If we assume that the slenderness ratios of concern are greater than 20 and the residual induction is on the order of 10,000 gauss, then the self demagnetizing force is in the tens of oersteds. This indicates that the optimum material has maximum residual induction at a demagnetizing force of a few tens of oersteds - but the coil requirements should be borne in mind. If a material is selected with large residual induction but requires a very large magnetizing force, then the penalty in coil weight may be very large.

The core material which gives a minimum weight combines minimum coercive force and maximum induction at a demagnetizing field of a few tens of oersteds.

Several secondary considerations should be borne in mind when evaluating alternative core materials -- considerations such as available size, special treatment required and mechanical properties.

Most of the useful core materials must be cold
worked to develop their magnetic properties, with
a 95% reduction in area being typical. Because of
this requirement, some materials are only available
in thin strip form or small diameter wire. These
materials also require annealing at approximately
600°C and must not be deformed after heat treatment.

A survey of 60 companies was conducted, and the
three most likely materials for this application
were selected. Remendur 38, P6 Alloy and Vicalloy
I were selected for maximum induction over three
ranges of demagnetizing fields. Remendur 38 has
the largest residual induction of these three
materials for demagnetizing fields less than 35
oersteds, while P6 Alloy has the greatest residual
induction for demagnetizing fields from 35 to 55
oersteds, and Vicalloy I extends the range to 100
oersteds. The properties of these materials are
summarized in Table 3-1, and the demagnetization
curves are shown in figure 3-1.

The choice of material for the magnetic core cannot
be made without considering the effects of self
demagnetization, and the requirements placed on the
magnetizing coil by the choice of core material.

3.2   Geometry Effects

As pointed out in 2.2, a permanent magnet has a self-
demagnetizing field which depends both on the geometry
and on the level of induction.

The effect of self-demagnetization is to increase the
volume of core material required to achieve a given
monent. This will also increase the coil requirements
because the magnetizing coil will enclose the volume
of core material, so a large core will require a large
magnetizing coil.

It is convenient to compare core materials graphically
by superimposing contours of constant residual moment
per cubed length (M/ℓ^3) on the B-H curves of the
materials. This is possible because equation 2-10
provides a link between the slenderness ratio (ℓ/D) and
the B-H coordinates, while equations 2-5 and 2-6 provide
a link between residual induction, core volume, and
dipole moment.

Combining 2-5, 2-6 and 2-10 we obtain

(3-26) $$B = 1.024 \times 10^{-4} \, \frac{p^2}{a} \, \frac{H^4}{\left(\frac{M}{\ell^3}\right)^3}$$

3-2

Table 3-1

	Remdender 38	P6 Alloy	Vicalloy	
Residual Induction (B_r)	16,500 (min.)	14,000	8,400	Gauss
Coercive Force (H_c)	$38^{\pm}10\%$	$58^{\pm}10\%$	240	Oersteds
Magnetizing Force (H_m)	100	250	1000	Oersteds
$\dfrac{Hm}{Hc}$	2.6	4.3	4.2	
Fullness $\dfrac{(BH)\ max}{Br\ Hc}$	.6	.61	.44	
Density (ρ)	8.2	8.16	8.2	grams/cm^3
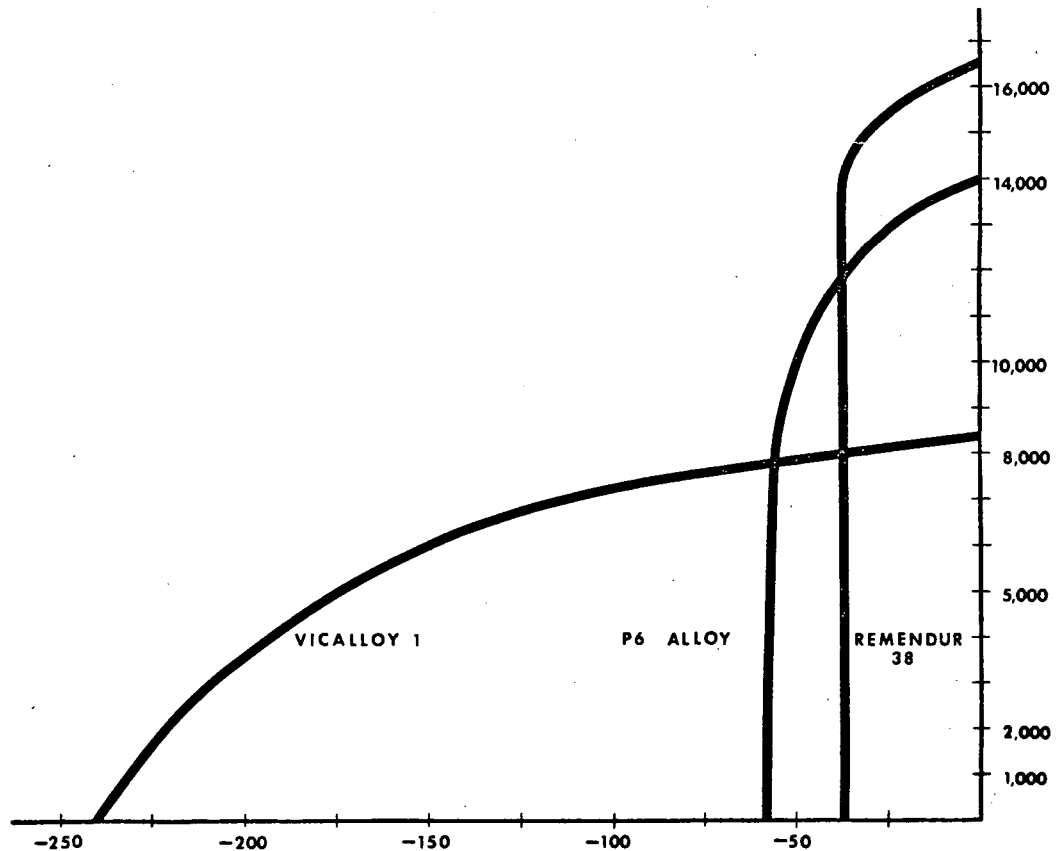
Figure 3-1   Demagnetization Curves for
Selected Materials

On substitution of appropriate geometry factors, this becomes

$$(3-27) \qquad B = 1.63 \times 10^{-3} \, \frac{H^4}{\left(\frac{M}{\ell^3}\right)^3} \qquad \text{for a square core cross section}$$

$$(3-28) \qquad B = 1.287 \times 10^{-3} \, \frac{H^4}{\left(\frac{M}{\ell^3}\right)^3} \qquad \text{for a round core cross section}$$

Figure 3-2 shows the B-H curves of 3 materials superimposed on contours of constant M/ℓ^3 from equation 3-27. These curves facilitate the inclusion of self-demagnetization effects when comparing core materials.

To illustrate the use of these curves, assume that a 10,000 p-cm magnet is desired with a maximum core length of 25.4 cm (10 inches). The ratio M/ℓ^3 for this case is .61, so we follow the M/ℓ^3 = .6 contour in figure 3-2 to find that Vicalloy I will require the largest core volume with a residual induction of 8100 gauss, P6 alloy will require an intermediate volume with 11,900 gauss and Remendur 38 will require the smallest volume @ 13,600 gauss. The minimum weight core for this example would be Remendur 38.
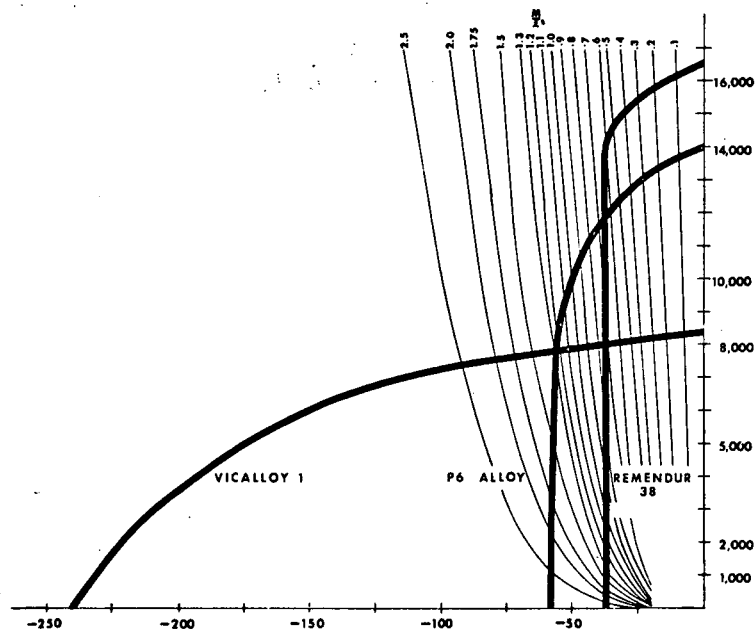
Figure 3-2

3.3   Interaction with Magnetizing Coil

The core design will affect the coil requirements in two ways: first, through the relationship between core and coil geometru; and second, by establishing the requirements on magnetizing force.

The core volume will determine the volume which must be supplied with a relatively uniform magnetizing force, so the size and shape of the core will obviously affect the volume of the magnetizing coil which enclose the core. Increasing core length will require more wire to cover the coil, while increasing core diameter will increase the average length of a turn. The magnetizing force which must be supplied by the magnetizing coil consists of the magnetizing force required to overcome self-demagnetizing effects and the net magnetizing force required to saturate the magnet. Once the material and required moment are specified, the total required magnetizing force will be a function of the slenderness ratio.

The interaction between peak coil power, coil weight and magnetizing force is seen by referring to figure 3-3. This figure shows the shape of the coil weight vs. peak coil power curve for a magnetic core of fixed size and material. The form of the equation plotted in Figure 3-3 is

$$\text{coil weight} = \frac{\frac{P}{H^2}\, C_2}{\frac{P}{H^2}^2 - \frac{P}{H^2}\, C_2 + C_3}$$

where P is peak power, H is peak magnetizing force and the C's are constants (see section 5.2).

Figure 3-3 shows that there is a threshold coil power for a given core design. Below this minimum coil power, it is not possible to design a coil which will magnetize the core. At large coil power the coil weight can be traded one for one with peak coil power, but at low coil power the increased coil build causes the coil weight to blow up.

Notice also that for a given coil design the peak power varies with the square of the magnetizing force. If the magnetic core is replaced with a material with twice the required magnetizing force, then the same magnetizing coil may be used but the required peak power will increase by a factor of four.
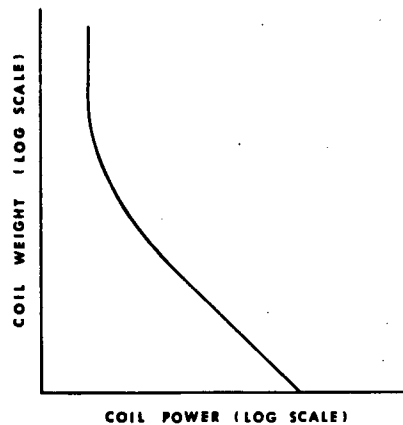
Figure 3-3 Coil Weight vs Peak Power

3.4   Scaling

Scaling of magnet designs is useful when it is desired
to produce a different magnet size from a given design
or when it is desired to evaluate a new design which
is an inconvenient size.

It is evident that if a magnet made of homogeneous
material has its dimensions scaled by a scale factor
s, then the maximum induction and magnetizing force
will remain constant. This is consistent with
equation 2-10 which says that self-demagnetization
effects depend only on shape and are independent of
size.

The spatial variation of magnetic field will have
the same shape, but measurement distances must be
scaled.

This is consistent with the scaling of all distances
and lineal dimensions.

The effective pole separation (effective magnet length)
will be scaled by s, but the effective pole strength
will be scaled by s^2. This comes about because the
pole strength is proportional to the cross sectional
area of the magnet rather than to the volume of
magnetic material.

Finally, the volume and weight of the material, the
dipole moment, and the energy required to magnetize
the material will all be scaled by s^3 -- which is
what would be intuitively expected.

3-6

Summarizing these scale factors, we find that: all dimensions and distances are scaled by s; volume, weight, and moment are scaled by s^3; and pole strength is scaled by s^2.

DESCRIPTION:

P-6 Alloy combines the advantages of a high hysteresis loss and low magnetizing force into one magnetic material. These two important features plus high residual induction and high permeability make P-6 Alloy especially useful for hysteresis motor applications.

This material is supplied in strip or wire form. The strip material is available in widths up to 4.0 inches and thicknesses from .005" to .100". Wire may be obtained in diameters ranging from .005" to .100". Magnetic certification is furnished with each shipment of material.

CHARACTERISTICS:

A. Composition:

Nickel	6.0%
Vanadium	4.5%
Iron	44.5%
Cobalt	45.0%

B. Magnetic Properties *(D.C.):

Maximum permeability	285
Peak magnetizing force, oersteds	250
Maximum induction, gausses	16,000
Residual induction, gausses	14,000
Maximum energy product ($B_d H_d$), gauss oersteds	550,000
Coercive force, oersteds	60
Hysteresis loss, joules/in^3/cycle	.37
peak H = 180 ampere turns/inch	

$\gamma = .655$

* At 25°C after age-hardening

C. Physical Properties:

Hardness prior to age-hardening	40 R.C.
Hardness after age-hardening	65 R.C.
Specific gravity	8.16
Density, lbs./cu. in.	0.285
Electrical resistivity, microhms-cm	30
Coefficient of expansion, in./in./°C	11.1 x 10^6

HEAT TREATMENT:

After stamping or coiling to the desired form, the material should be either vapor degreased or emulsion cleaned prior to final age-hardening heat treatment. The material should then be placed in an electric furnace with hydrogen or cracked gas atmospheres and held at 600°C for two hours. This final heat treatment is necessary to obtain the desired magnetic characteristics of the material.

DESIGN CONSIDERATIONS:

A. General:

P-6 Alloy is capable of supplying superior hysteresis characteristics in designs where a maximum of 100 oersteds magnetizing force is available.

The high aging temperature of P-6 alloy allows it to be subjected to a much higher operating temperature than the chrome, tungsten and cobalt magnet steels without resulting in subsequent aging. The material is magnetostrictive, however, and laminations should therefore be stacked and contained with a minimum of stress applied to the material. Figures 1, 2 and 3 respectively show the hysteresis loop, energy product and magnetization curves; the hysteresis loss vs. maximum flux density; and the hysteresis loss vs. magnetizing force for fully aged P-6 Alloy.

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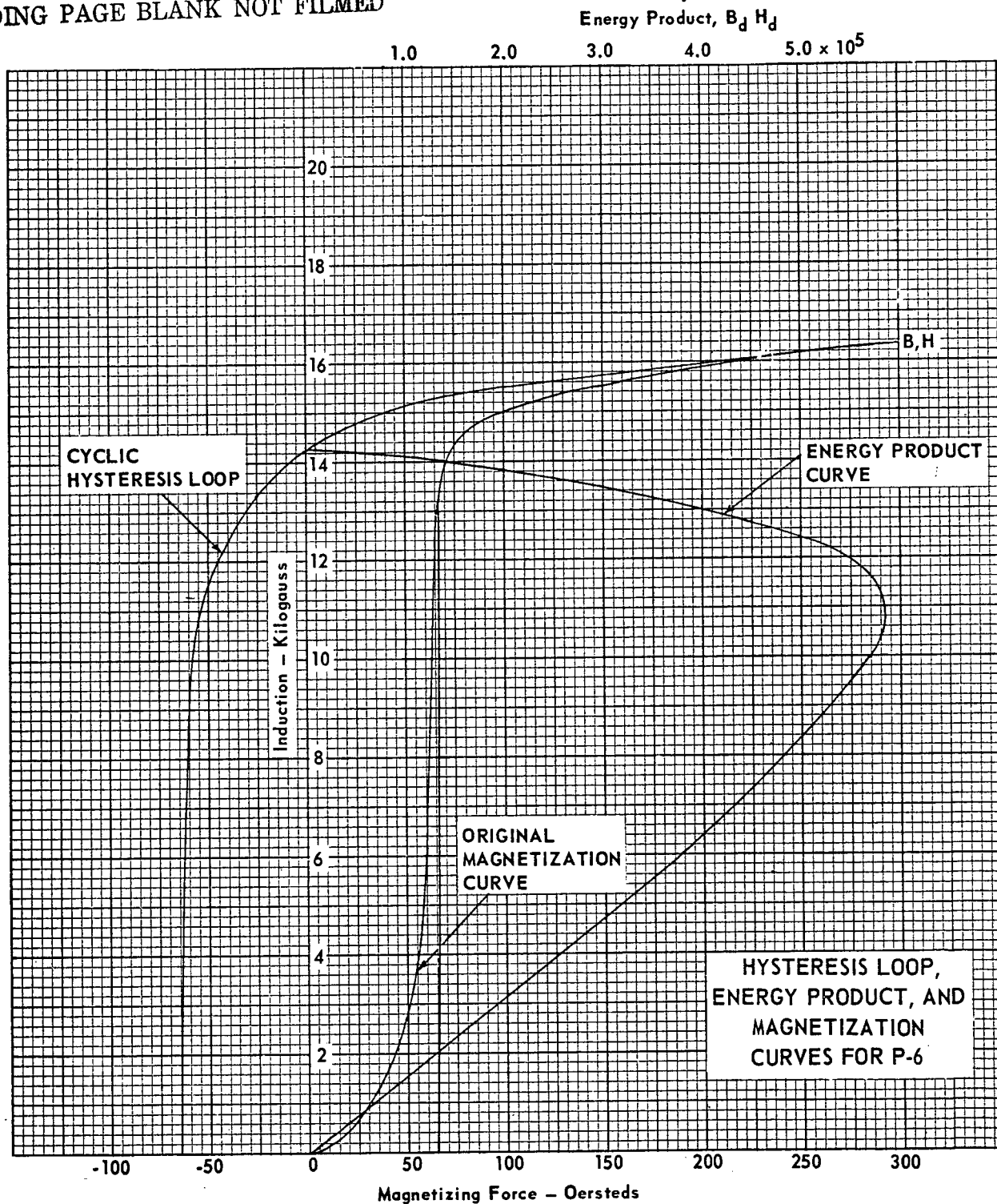
FIGURE 1

Energy Product, $B_d H_d$

CYCLIC HYSTERESIS LOOP

ENERGY PRODUCT CURVE

B,H

ORIGINAL MAGNETIZATION CURVE

HYSTERESIS LOOP, ENERGY PRODUCT, AND MAGNETIZATION CURVES FOR P-6

Induction – Kilogauss

Magnetizing Force – Oersteds

Typical Hysteresis Loop and Energy Product for Remendur 38 Heat Treated at 1100 F for 2 Hours.

ENERGY PRODUCT — GAUSS OERSTEDS x 10^5

HYSTERESIS LOOP

ENERGY PRODUCT

INDUCTION — KILOGAUSS

MAGNETIZING FORCE OERSTEDS
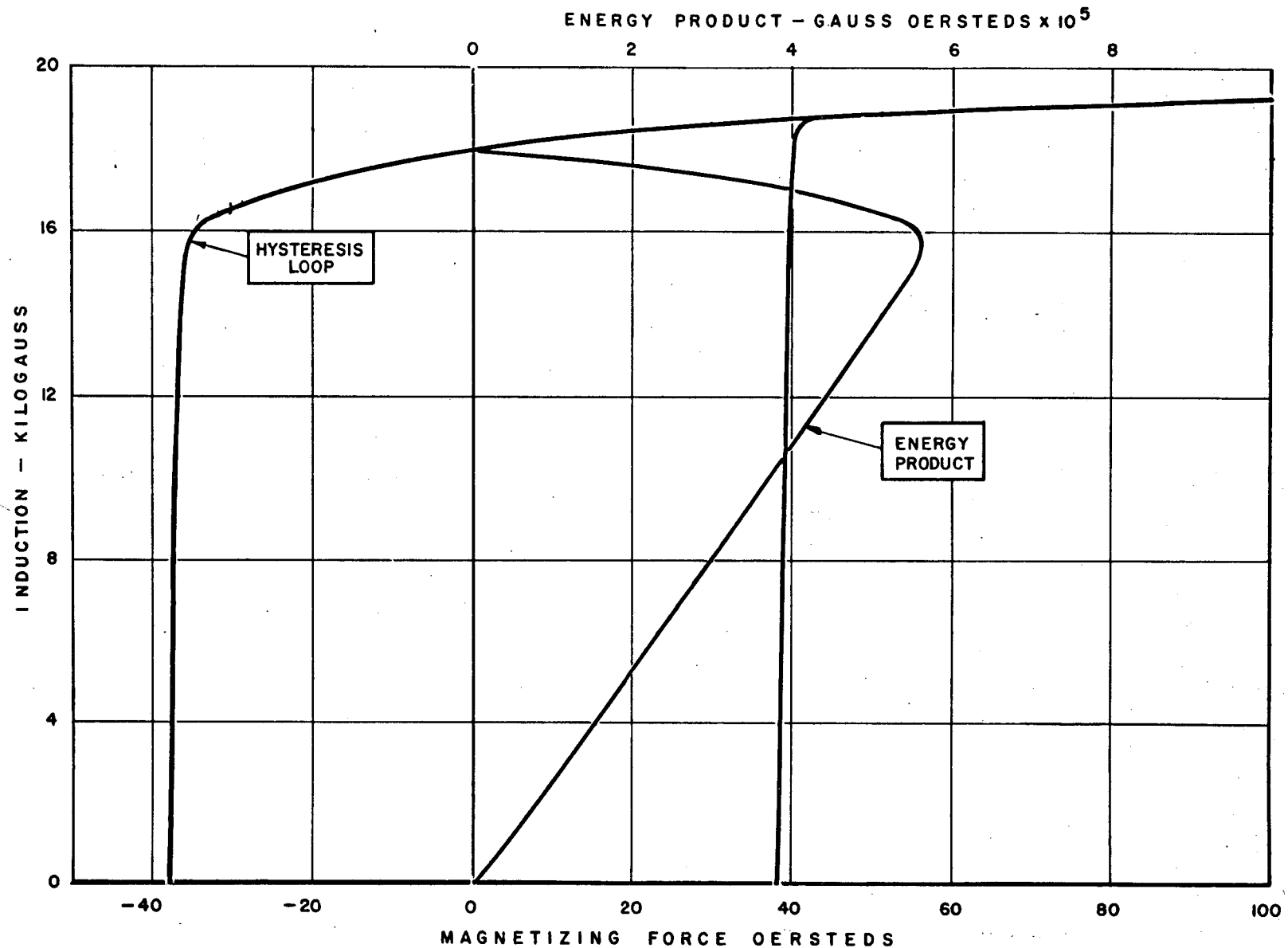
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4.0 MAGNETIZING COIL WEIGHT

In this section we treat the problem of determining the wire weight for a magnetizing coil.

4.1 Derivation of Coil Weight Equations

The resistance of a length of wire in terms of its resistivity and dimensions is given by

$$(4\text{-}29) \qquad R = \frac{4\ell_w}{\pi d^2}\rho$$

or the resistance for a specific size and material is given by

$$(4\text{-}30) \qquad R = r\ell_w$$

where r is lineal resistivity - e.g. resistance per thousand feet of wire length. Similarly, the weight of a length of wire in terms of wire dimensions and density is given by

$$(4\text{-}31) \qquad W_w = \ell_w \; \frac{\pi d^2}{4} \; \delta_w$$

or an expression similar to 4-30 can be written for wire weight.
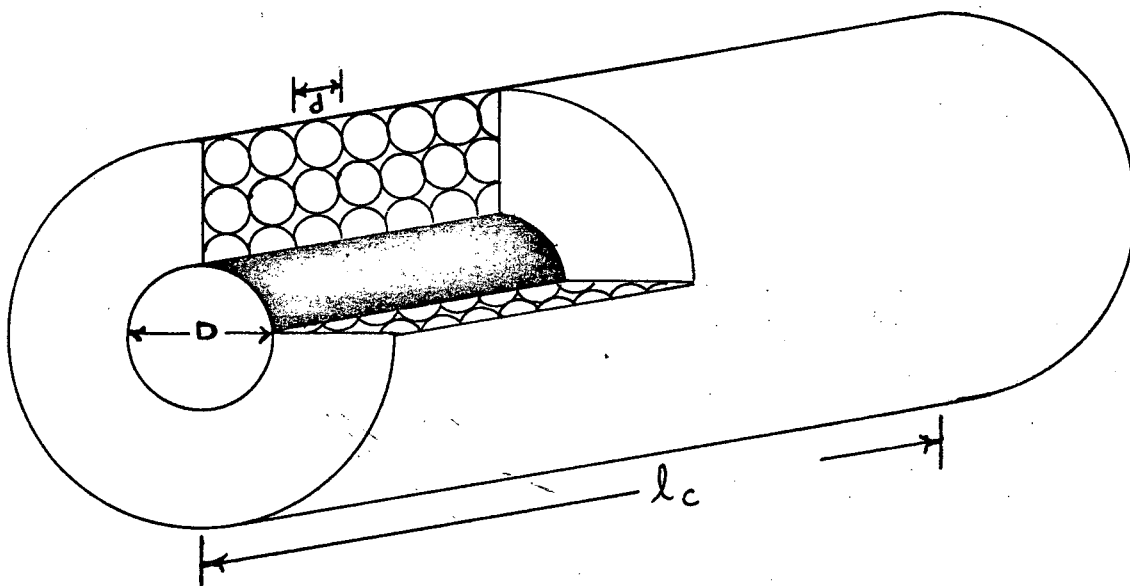
Figure 4-1 Coil Dimensions

4-1

The length of wire required to wind a coil as shown in Figure 4-1 can be approximated in three steps. The first order approximation is that wire length equals the product of the perimeter of the cross section and the number of turns.

(4-32)
$$\mathcal{l}_w = (pD) \ n$$

The second order approximation corrects for the effect of coil build - i.e., for multilayer construction. Because alternate coil layers will have opposite pitch, the diameter of the coil will increase by two wire diameters for each additional layer of wire. The length of one turn in the jth layer will then be

$$(pD) + 2\pi d \ (j - \frac{1}{2})$$

and the length of all turns in λ layers is approximately

(4-33)
$$\mathcal{l}_w = n \ (pD) \ \left[1 + \frac{\pi}{p} \ \frac{\lambda d}{D} \right]$$

It is of value to note that (π/p) is nearly 1, so the second term in the brackets is nearly the ratio of coil build to coil diameter.

The next refinement is very rarely needed unless the wire diameter approaches the core diameter. The additional wire length due to the pitch of the winding is simply the length of the coil multiplied by the number of coil layers. The expression for the length of wire in a coil of λ layers is

(4-34)
$$\mathcal{l}_w = n \ (pD) \ \left[1 + \frac{\pi}{p} \ \frac{\lambda d}{D} \right] \ + \lambda \mathcal{l}_c$$

This has been found to give ±1% accuracy for carefully wound coils. From this we can compare the amount of wire used for different cross-sectional shapes. For instance, consider two cores with equal cross sectional area. One having square cross section with side w, and the other being circular with diameter D.

For the square:

$$\ell_{w1} = 4nw + n\pi\lambda d$$

For the circle:

$$\ell_{w2} = nD + n\pi\lambda d$$

Invoking the equal area constraint we have for the ratio of the wire lengths

$$\frac{\ell_{w1}}{\ell_{w2}} = \frac{\frac{4}{\pi}\sqrt{\frac{A}{d^2}} + \lambda}{\sqrt{\frac{4A}{\pi d^2}} + \lambda}$$

This ratio is plotted vs (A/d^2) with λ as a parameter in the graph of Figure 4-2. This shows that the penalty for using a core with square core cross section rather than a round cross section varies from 13% when a single layer of fine wire is wound on a large core to 26% when many layers of heavy wire are wound on a small core.

Included in the figure is a nomograph to convert cross sectional area and wire gauge to (A/d^2). The nomograph assumes double formvar insulation, so the diameter used when constructing the nomograph was greater than the bare wire diameter listed in wire tables.

The use of the nomograph is illustrated by taking a core with 0.5 square inch cross sectional area with 10 layers of 24 AWG. The intersection of the 24 AWG line and the 0.5 in^2 line gives an (A/d^2) of 1005. Following the (A/d^2) line down to the 10 layer curve, we find the length ratio is 1.160. For this example 16% more wire is needed for a square cross section than for a circular cross section of equal area.

Figure 4-3 facilitates comparison of wire length with circular cross sections when only the wire diameter and the number of layers is changed. If the same core is used, then the periphery (pD) will not change and if the number of turns is also constant, then the relative wire length is
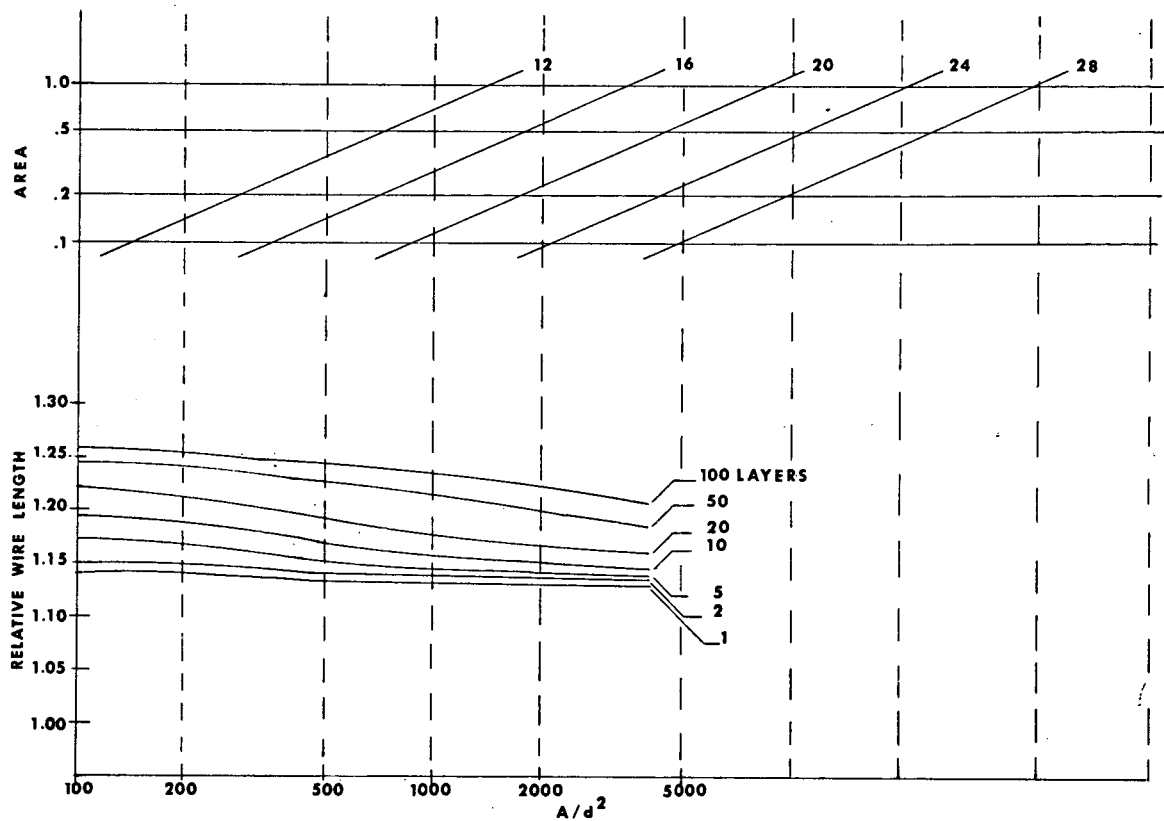
Fig. 4-2 Wire length for Square Core Relative to Wire length
for a Circular Core

(4-36) $$\frac{\ell_w}{npD} = 1 + \frac{\lambda d}{D}$$

from equation 4-33.

To illustrate the use of Figure 4-3, assume that a
coil is to be designed with 1000 turns of copper wire
and 5Ω resistance on an 8 inch long by .5 inch diameter
round core.

4-4
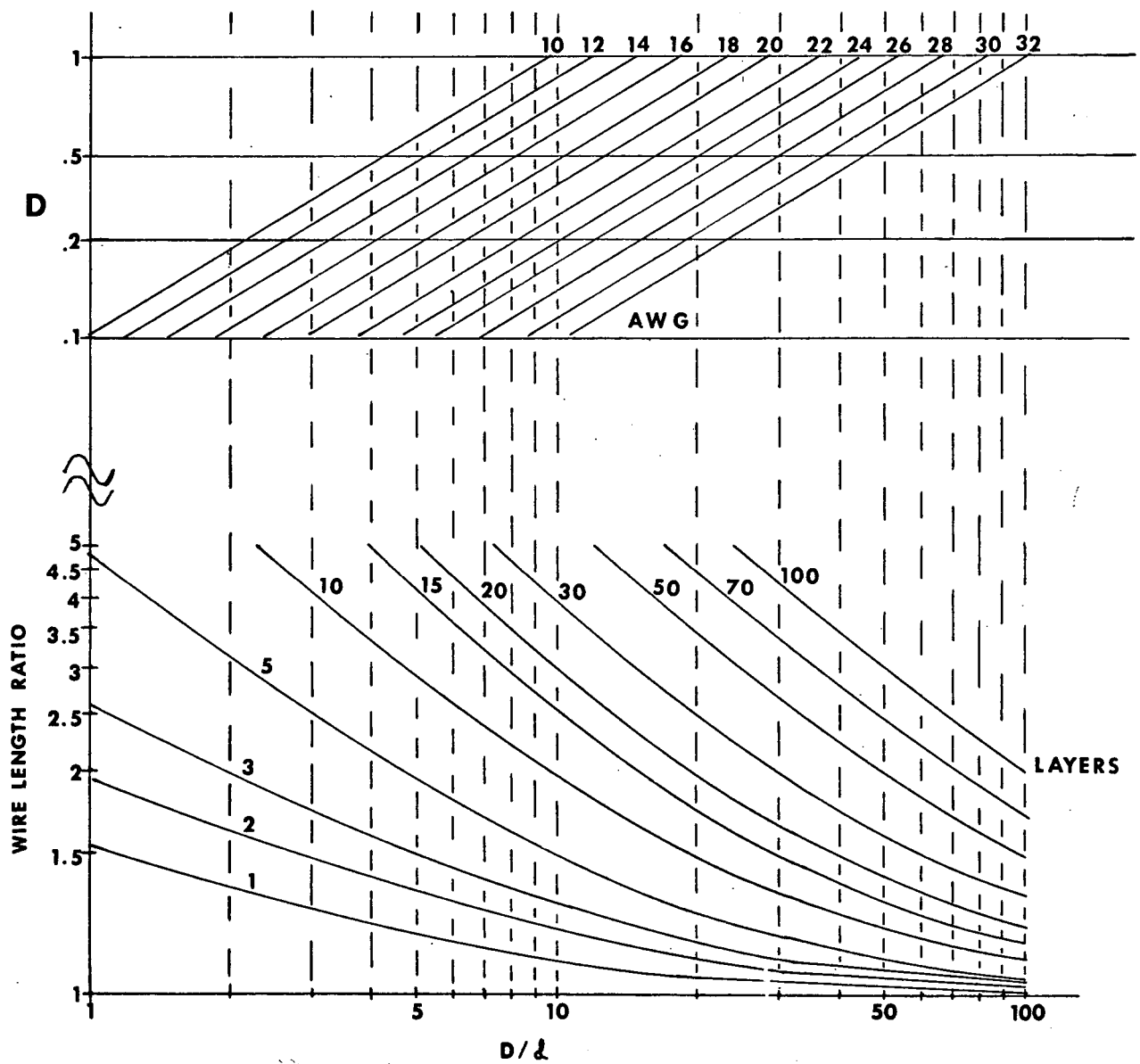
Fig. 4-3   Effect of Coil Build on Wire Length

4-5

WIRE TABLE

AWG	Diameter in Inches		Ohms per 1000 feet	
	Bare	Double Formvar	Aluminum	Copper
10	.1019	.1055	1.637	.9989
12	.0808	.0842	2.603	1.588
14	.0641	.0673	4.139	2.525
16	.0508	.0538	6.584	4.016
18	.0403	.0431	10.47	6.385
20	.0320	.0346	16.64	10.15
22	.0253	.0277	26.46	16.14
24	.0201	.0223	42.08	25.67
26	.0159	.0179	66.90	40.81
28	.0126	.0145	106.4	64.90
30	.0100	.0116	169.2	103.2
32	.0080	.0094	269.0	164.1

TABLE 4-1

The periphery of the core is 1.57 inches (.5π),
so 1000 turns will require at least 1570 inches
or 131 feet.

The lineal resistivity is then on the order of 5Ω
per 131 feet or 38Ω per 1000 feet. As we expect
to use more than 131 feet of wire because of coil
build, we look up the wire size with a slightly
lower lineal resistivity in Table 4-1 and find 24
AWG with 25.67Ω per 1000 feet.

Next we approximate the number of layers by taking
the length of a single layer coil (number of turns
multiplied by wire diameter) and divide by the
desired coil length.

$$\lambda \simeq \frac{1000 \text{ turns} \times .0223 \text{ inch/turn}}{8 \text{ inch}}$$

$$\lambda \simeq 2.8$$

With the preliminaries completed, we use the wire nomograph at the top of figure 4-3 to find that (D/d) for a .5 inch diameter core and #24 AWG is 22.4.

Next, enter figure 4-3 at (D/d) = 22.4 and 3 layers. The wire length ratio is 1.13, so the wire length is

$$\ell_w = 1.13 \times 131 \text{ feet}$$

$$\ell_w = 147 \text{ feet}$$

and the resistance of the coil is

$$R_c = 147 \text{ feet} \times 25.67\Omega/1000 \text{ feet}$$

$$R_c = 3.77\Omega$$

If this coil is wound with full layers rather than allowing the last layer to partially cover the coil, then the coil length will be

$$\ell_c = \frac{1000 \text{ turns} \times .0223 \text{ inch/turn}}{3 \text{ layers}}$$

$$\ell_c = 7.43 \text{ inch/layer}$$

Following this procedure we can quickly evaluate alternative wire sizes, wire materials and determine the fraction of the coil length occupied by the last layer.

From Figure 4-2, a considerable savings in wire length results by replacing a square core with one of circular cross section. It is inconvenient to fabricate a magnet with a true circular cross section from stacked laminations, but a circular cross section may be approximated by varying the widths of the laminations.

If many laminations are used, the cross sectional shape may be approximated by a regular polygon of (2N+2) sides - where N is the number of laminations. This accounts for 2N exposed edges of the laminations and the two exposed sides.

This polygonal approximation implies a variation in the thickness of the laminations because the sides of the polygon are not perpendicular to the width of the laminations. The ratio of the minimum thickness to the maximum thickness is

$$\frac{\check{T}}{\hat{T}} = \frac{\ell \sin \frac{\pi}{N+1}}{\ell}$$

$\check{T}$ = minimum lamination thickness

$\hat{T}$ = maximum lamination thickness

ℓ = length of a side of the polygon

N = number of laminations

With this limitation in mind we proceed as follows.

The perimeter and area of the polygon are given by

$$Area = (2N+2) \frac{D^2}{4} \tan \frac{\pi}{2N+2}$$

$$Perimeter = (2N+2) D \tan \frac{\pi}{2N+2}$$

D = diameter of inscribed circle

N = number of laminations

2N+2 = number of sides to polygon

But if N is greater than 5, then

$$(2N+2) \tan\left(\frac{\pi}{2N+2}\right) = 12 \tan (15°)$$

$$= 1.023\pi$$

So, within 2%, the area and perimeter of the polygon are the same as that of the inscribed circle.

From this we conclude that the advantage in wire length for circular cross section can be gained by stacking as few as five laminations if the widths are cut to approximate a circular cross section.

4.2 Conductor Materials

At present, the only practical conductor materials are aluminum and copper, but should another material be considered, some of the most desirable properties are given here.

The conductor characteristics of prime importance are density and resistivity. Quite obviously, the conductor material should be selected for minimum density to minimize coil weight, and should be selected for minimum resistivity to minimize power. Unfortunately, these two characteristics do not necessarily go hand in hand, and the optimum material will depend on the specific application. In the case of a planar air coil with little build, the power and weight may be traded one for one. In this case the product of density and resistivity becomes a meaningful measure of conductor quality. But when the coil build is appreciable, this is no longer a useful guide.

It is possible to predict the crossover point where comparison of the resistivity-density product of two materials will no longer indicate the conductor material for a minimum weight coil, but this must be done for a specific core geometry and specific core material. (See section 4.3)

Should a conductor material other than aluminum or copper be considered, several other properties besides resistivity and density must be considered. Copper and aluminum have well known properties and any other material will require extensive effort to become established. The properties which must be established include flexibility, mechanical stability under winding pressure and tension, susceptibility to creep, effects of impurities, and other odd properties such as the tendency of silver to migrate through dielectric materials or the growth of crystals from the base metal (whiskers).

4.3   Coil Power

The magnetizing coil for a given magnetic core may be designed for a wide range of peak magnetizing power, with coil weight decreasing as peak power is increased. The lower limit on coil power is fixed by the magnetic core design and the coil conductor material. These effects are easily demonstrated by taking specific examples.

The minimum power effect is illustrated by examining the effect of adding more layers of wire to the magnet assembly in Figure 4-4 which is a single layer coil wound with wire whose diameter is one tenth the core diameter. The change in coil resistance due to the additional coil layers is proportional to the increase in wire length.

The original layer of wire will have a mean length per turn of (d + 10d), and the mean turn length in a second layer will $\pi d(10 + 3)$. The mean diameter of subsequent layers will increase by two wire diameters for each additional layer so the mean length of a turn in the $_n$th layer is $\pi d(9 + 2\lambda)$.
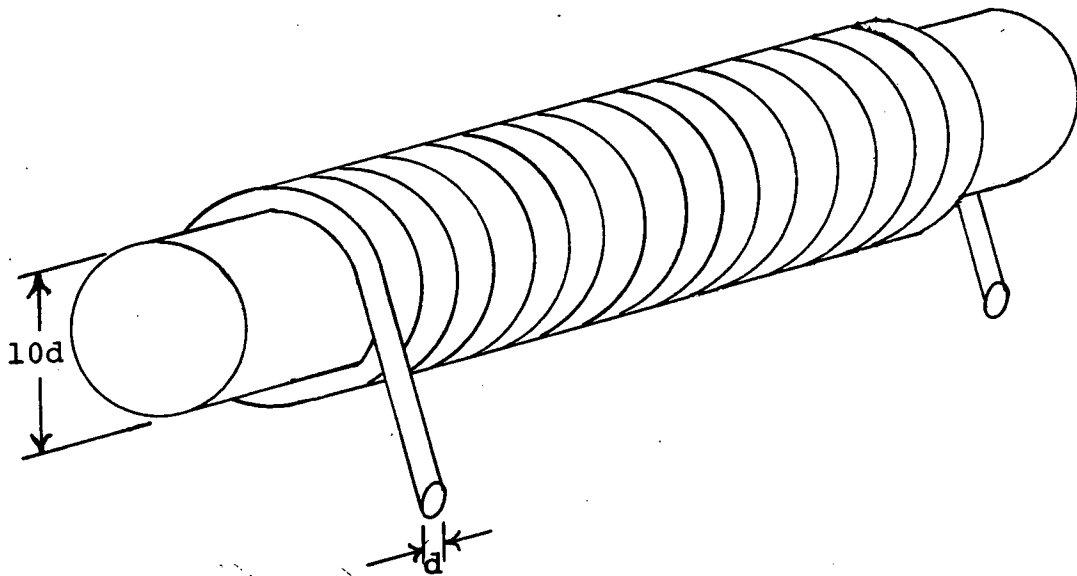
Figure 4-4

Using

$$P = i^2 R$$

and letting subscripts denote the total number of layers, the power in the original coil is given by:

$$P_1 = (i_1)^2 R_1$$

Now when we add a second layer of wire we will have doubled the number of turns so we must halve the current to hold the magnetizing field strength constant.

$$P_2 = \left(\frac{i_1}{2}\right)^2 R_2$$

$$= \left(\frac{i_1}{2}\right)^2 R_1 \left[1 + \frac{\pi d\ (10+3)}{\pi d\ (10+1)} \right]$$

$$P_2 = \frac{(i_1)^2 R_1}{2} \times 1.09$$

For a three layer coil we have

$$P_3 = \left(\frac{i_1}{3}\right)^2 R_3$$

$$= \left(\frac{i_1}{3}\right)^2 R_1 \left[1 + \frac{\pi d\ (10+3)}{\pi d\ (10+1)} + \frac{\pi d\ (10+5)}{\pi d\ (10+1)} \right]$$

$$P_3 = \frac{(i_1)^2 R_1}{3} \times 1.18$$

and for a λ layer coil the power is

$$P\lambda = \frac{(i_1)^2 R_1}{\lambda}\ \frac{10+\lambda}{10+1}$$

Now we have obviously reduced the power requirements, but not as rapidly as we have increased the total number of turns. The limiting power with an infinite number of coil turns (and layers) will be given by

$$\lim_{\lambda \to \infty} P\lambda = (i_1)^2 R_1 \frac{1}{D/d + 1}$$

This would seem to indicate that there may be an advantage to using fine wire so that the limiting power would be less. But the actual effect is only to increase the power for a single layer coil so that a greater reduction in power is possible.

Notice in the previous illustration that coil weight increases more rapidly than the number of turns because of the increased length of a turn. By the same line of reasoning used in determining the coil resistance, the coil weight is

$$W\lambda = W_1 \; \lambda \; \frac{10 + \lambda}{10 + 1}$$

and the product of coil weight and power becomes

$$P\lambda W\lambda = P_1 W_1 \; \frac{10 + \lambda}{10 + 1}^2$$

or to give specific examples

$$P_2 W_2 = 1.19 \; P_1 W_1$$

$$P_3 W_3 = 1.40 \; P_1 W_1$$

$$P_4 W_4 = 1.62 \; P_1 W_1$$

Figure 4-5 shows the effect of increasing the number of coil layers on the coil weight, coil power and on the coil weight - power product. These results are not restricted to this specific geometry, but can be generalized to show that the minimum magnetizing power is independent of the wire diameter.
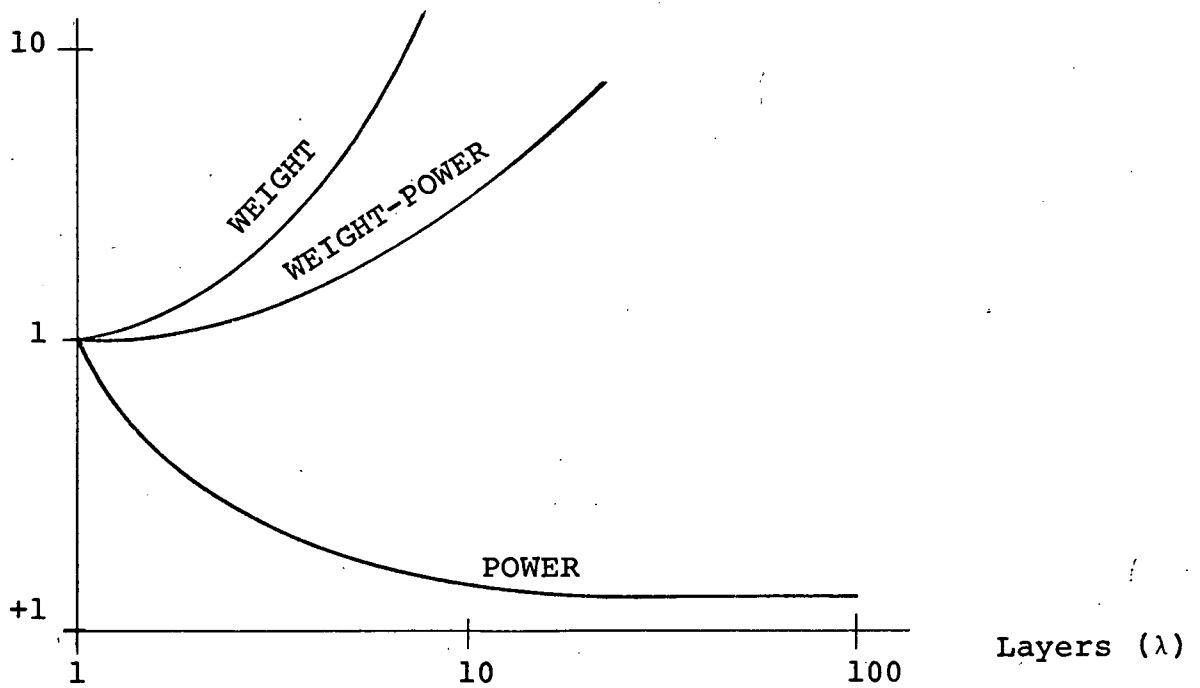
Figure 4-5   Variation of Coil Parameters with Coil Layers

In the limit when extremely large coils are used, the actual shape of the core cross section becomes immaterial, and the limiting power is a function of the magnetic field strength required, the length of the coil and the resistivity of the conductor. By combining equations 2-13, 4-29, 4-31 and 4-33 we may solve for the limiting power with an infinite amount of conductor. The limiting power is given by

$$P_O = 2.81\ H_{pk}^2\ \ell c'$$

which for aluminum and copper conductor materials becomes

$$P_{al} = 7.95 \times 10^{-6}\ H_{pk}^2\ \ell c$$

$$P_{cu} = 4.85 \times 10^{-6}\ H_{pk}^2\ \ell c$$

where a reasonable estimate of insulation thicknenn has been included.

These results point to another effect due to coil build. When a coil is constructed of aluminum and another coil is constructed of copper, with the same wire diameter, number of layers and turns, the aluminum coil has one third the weight because aluminum is less dense. The copper coil requires 60% as much power as the aluminum coil because copper has a greater conductivity. We might therefore expect to be able to construct an aluminum coil that requires the same power as the copper but with less weight. But coil weight and power do not trade one for one, so an aluminum coil is lighter only when the power is above some threshold value.

This threshold power can be determined by setting the weight of two coils equal and solving for the power. The resulting threshold power for materials denoted by subscripts 1 and 2 is approximately

$$P_{1-2} = 2.81 \times H_{pk}^2\ \ell c\ \ 2\ \frac{\dfrac{1}{\delta 1} - \dfrac{1}{\delta 2}}{\dfrac{1}{\rho 1 \delta 1} - \dfrac{1}{\rho 2 \delta 2}}$$

And this threshold power for the most common choice
of aluminum versus copper conductor is

$$P_{al-cu} = 26 \times 10^{-6} \, H_{pk}^{2} \ell c$$

For a 10,000 p-cm magnet 25 cm long and requiring 150
Oersteds, this threshold power is on the order of
fourteen watts.

The peak coil power for a permanent magnet is useful
for determining the weight of the magnetizing coil and
for determining the circuit requirements for the coil
driver. However the peak coil power is not a valid
basis for comparing the power of different systems un-
less the time duration of the power drain is considered.

The charging energy -- or average power during a charg-
ing cycle can be shown to be independent of the coil
resistance. This can be seen by considering a series
RL circuit (Fig. 4-6). When the switch is closed the
current willfollow the familiar exponential rise as
shown in Fig. 4-6.

If the circuit is interrupted when the current reaches
some portion of the maximum possible $\dfrac{V_{o}}{R}$,
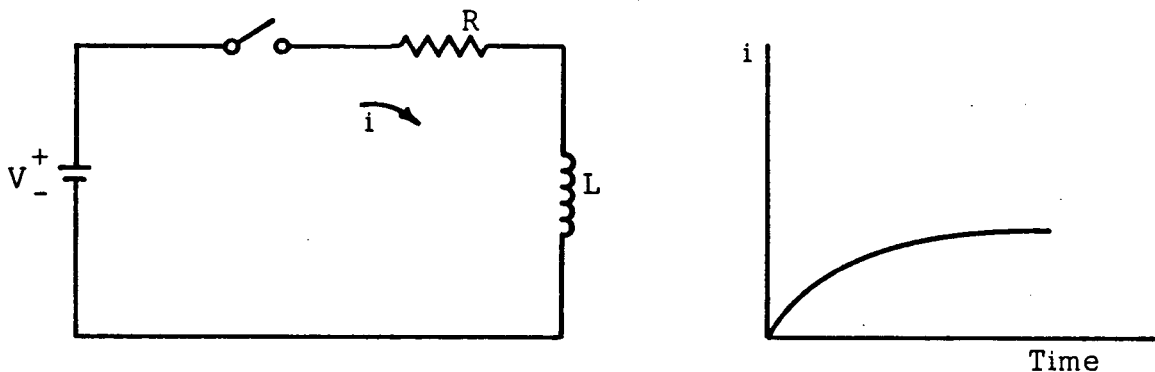
Figure 4-6

then the energy supplied will be the product of the
supply voltage and the average current supplied to the
circuit. If the resistance is then doubled, the supply
voltage must be doubled to maintain the same maximum
current. Now, when the switch is closed the current
will rise twice as fast because the L/R time constant
has been halved. If the switch is opened at the same
current level as before, then the peak current will be
the same as before, but the energy supplied to the
circuit has not changed.

This same line of reasoning can be extended to show that the energy required to apply a given magnetic field to a given core is independent of the coil parameters. This, of course, assumes that the spatial distribution of the magnetic field is the same for all coils. A very large diameter coil will obviously have more leakage than a small diameter coil of the same length, but when treating relatively small coils the energy required to charge a magnet is independent of coil turns, wire size and conductor resistivity.

4.5   SCALING

When the magnetic core is scaled (see 3.5) to produce a different magnet size from a given design, the coil dimensions must also be scaled. In this section we take a scale factor s, scale all coil dimensions, and determine the effects of this scaling.

If the coil length and wire diameter are scaled by the same factor, then the number of turns which will fit on one layer will be constant. Similarly, the number of layers is constant when the wire diameter and coil built are scaled by the same scale factor.

The length of a turn in any particular layer is scaled by s, and consequently the wire length is scaled by s.

The resistance of the coil varies directly with the wire length and inversely with the square of the wire diameter, so the coil resistance is scaled by s^{-1}.

For the magnetic field strength to remain constant, the current must be scaled by s. This can be seen from equation 2-13.

$$H = \frac{.4\pi n_1 i_1}{\ell_1} = \frac{.4\pi n_2 i_2}{\ell_2}$$

$$\frac{.4\pi n_1 i_1}{\ell_1} = \frac{.4\pi n_1 i_2}{s\ell_1}$$

$$i_2 = s i_1$$

The scale factor for peak power can now be readily deter-
mined from

$$P = i^2 R$$

by using the scale factors for current and resistance.
The scale factor for peak power is s. The energy required
to charge the magnet is proportional to the volume of the
coil and core - e.g. to s^3. Thus peak power scales by s
and magnetizing time by s^2.

The last scale factor to consider is for voltage. This
is easily found by taking the product of the scale factors
for current and resistance, with the result that voltage
is unchanged.

Another simple scaling method is possible - the coil
dimensions may be scaled by s, but the wire diameter is un-
scaled. This method of scaling gives a scale factor of s^3
for coil resistance and wire length. The scale factor for
peak power, number of turns per layer and number of layers
is s. The coil voltage is scaled by s^2 and peak current
by s^{-1}. The magnetizing time is scaled by s^2, and peak
power by s so that magnetizing energy is again scaled by
s^3.

There are two limitations to coil scaling: fine wire
usually has relatively heavy insulation; wire diameter is
quantized at approximately 20% intervals. Neither of
these is a serious objection and only a minor adjustment to
a scaled coil design is ever needed in practice.

5.0   MAGNET WEIGHT

5.1   Core Weight

In section 2.2, it was pointed out that short magnets generate a self-demagnetizing field which partially demagnetizes the magnet in the absence of an externally applied magnetizing force. This geometry effect was used in section 3.2 to find the maximum residual induction in the magnetic core as a function of the moment and core length.

Because the dipole moment, induction and volume of core material are related by equations 2-5 and 2-6, and by assuming 30% leakage, we may write for the core volume

(5-37) $$\text{Volume} = \frac{4\,M_r}{0.7 B_r}$$

where M_r and B_r are the residual moment and the residual induction respectively. The weight of the magnetic core is simply the product of core volume and the density of the core material, so data points from 3-2 may be used to generate curves of core weight vs residual moment vs core length.

Figure 5-1 gives core weight as a function of core length for a 1000 pole-cm magnet. As might have been expected, these curves flatten out for long magnets where the self-demagnetizing force becomes small.

This curve may be used for other magnet sizes by scaling (see section 3.5 or 5.4). The weight will scale directly with the moment and the length with the cube root of the moment. To illustrate, consider the problem of finding the minimum core weight for a 12,000 pole-cm P6 magnet in 25cm of length. The dipole moment is scaled by a factor of twelve, so the length is scaled by the cube root of twelve or 2.29. The length scaled down to 1000 pole-cm is 10.9cm (25cm divided by 2.29,) so we enter figure 5-1 at 10.9cm. The indicated weight is 12.9 grams for P6 material, which must be scaled by twelve. The minimum weight for a 12,000 pole-cm magnet of P6 material in 25cm of length is 155 grams.

WEIGHTS OF 1000 P—CM MAGNETS

FIGURE 5—1   WEIGHT VS LENGTH OF 1000 P—CM MAGNETS
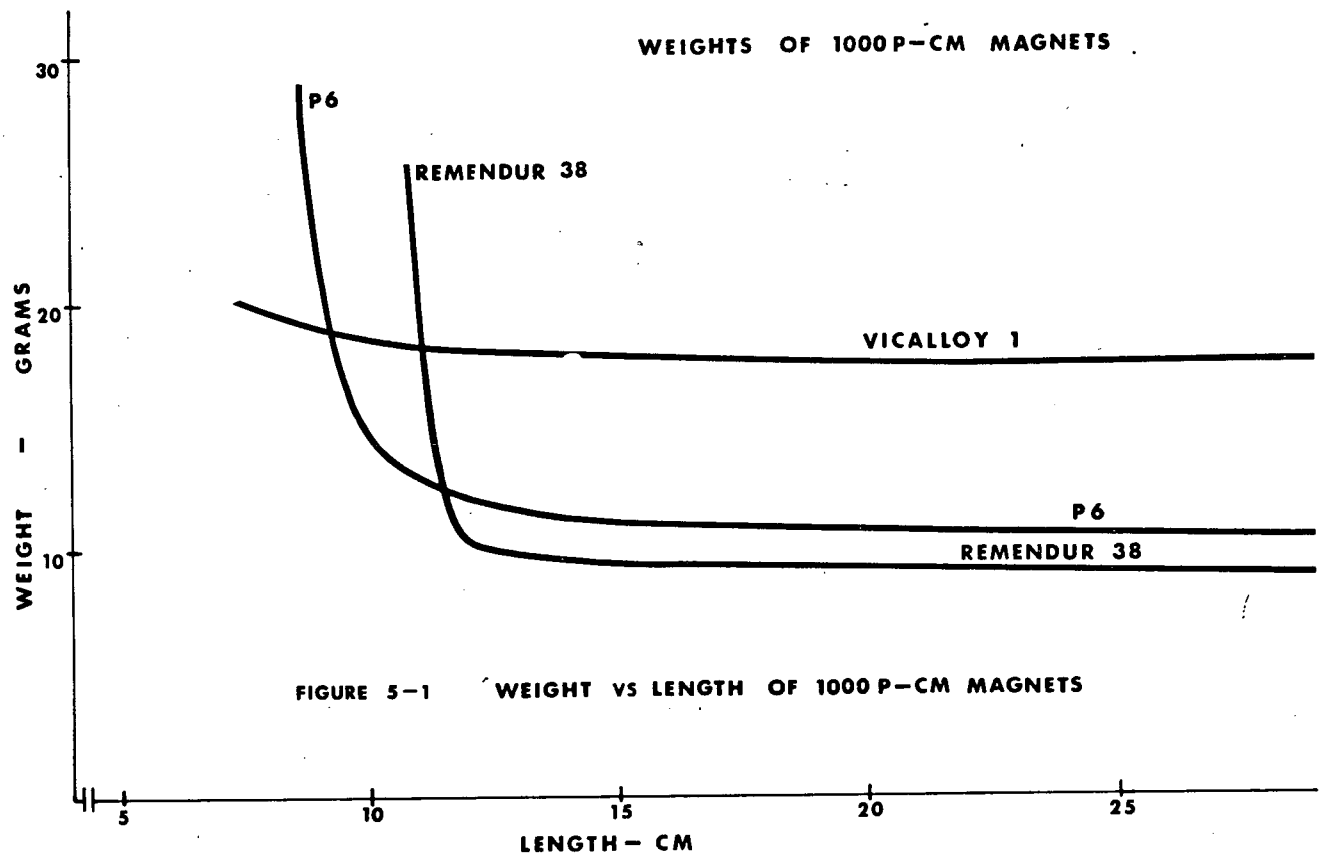
Figure 5-1

5.2 Coil Weight

The weight of the magnetizing coil is determined by the magnetic core material, the required dipole moment, magnet length and the peak power.

Specification of the magnetic core material, dipole moment and magnet length will determine the slenderness ratio and the core volume. These in turn determine the peak magnetizing force necessary to satisfy the material requirements and to overcome the self-demagnetizing force (section 2.2). The coil weight is an increasing function of core volume and peak magnetizing force, but is a decreasing function of coil power.

The peak coil power cannot be reduced beyond a threshold power because coil losses increase rapidly with coil size (section 4.3). The minimum coil power is given by

(5-38)
$$P_0 = 2.81 \ H_{pk}^2 \ \ell_c \ \rho$$

Several expressions for the coil weight may be derived from the information in chapters 3 and 4. One convenient form giving the coil weight relative to the core weight is

(5-39)
$$\frac{W_{wire}}{W_{core}} = \frac{\dfrac{\rho^2 c}{a} \ \dfrac{\delta_{wire}}{\delta_{core}}}{4.437 \ \dfrac{P}{P_0} - \dfrac{80}{9} + 4.452 \ \dfrac{P_0}{P}}$$

This equation with the numerator set equal to one is plotted in Figure 5-2. Notice that for large
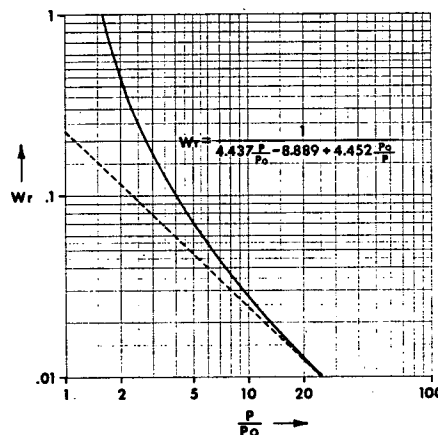
Figure 5-2 Variation of Coil Weight with Peak Power

peak power the curve is asymptotic to $(4.437 \ P/P_0)^{-1}$.

5-3

5.3   Magnet Assembly Weight

The weight of the total magnet assembly consists not only of the weight of the core and coil, but also includes the weight of the housing, potting or encapsulation, connector(s), and any internally mounted sensors for temperature, moment, etc.

The sensors will generally be very small, light devices and their weight will be insignificant. But the weight of the housing, potting and connector will depend on the specific circumstances and can be expected to increase the total weight by at least 25%.

In view of the number of variables, it is more convenient to find the weight of the core and the weight of the foil separately as detailed in sections 5.1 and 5.2.

5.4   Scaling

When scaling an extant magnet design to obtain a different size, there are two principle limitations. The most troublesome of these is that wire is available only in discrete sizes, so exact coil scaling will not usually be possible.

The second important limitation is the effect of scaling on the magnetic properties of the core materials. As discussed in 3.5, this is not a serious limitation if the magnet core is fabricated of stacked limitations.

Table 5-1 lists most parameters of interest with their scale factors for a magnet with all dimensions scaled by a factor s. These scale factors are exact for materials whose properties do not change with size - in spite of the heuristic development in sections 3.5 and 4.5.

TABLE 5-1 MAGNET ASSEMBLY SCALE FACTORS

	Fixed Voltage Scaling	Fixed Wire Size Scaling
Moment (M)	s^3	s^3
Total Weight	s^3	s^3
Magnetizing Energy	s^3	s^3
Core length ( )	s	s
Core diameter (D)	s	s
Distance for equal remote field (r)	s	s
Magnetic field strength (H)	1	1
Magnetic induction (B)	1	1
core weight	s^3	s^3
coil length (c)	s	s
coil turns ( )	1	s^2
côil layers ( )	1	s
turns per layer	1	s
wire diameter	s	1
wire length	s	s^3
coil weight	s^3	s^3
coil current	s	s^{-1}
coil resistance	s^{-1}	s^3
voltage	1	s^2
power	s	s

where s is a dimensionless scale factor.

6.0 Magnet Design Curves

The following curves can be utilized to estimate the size, weight, and power of chargeable permanent magnets. The design curves are based on the elliptical geometry described in the next section.

6.1 Core Weight and Length

The size and weight of the magnetic core for a wide range of dipole moments can be determined from Figures 1 and 2.

6.2 Coil Weight

The weight of the magnet coil will depend on the peak power used to change the magnetic state. By peak power is meant the power required to just saturate the magnet. This dependency can be seen by performing a mental experiment. If half the turns are stripped off the coil, the resistance, wire length, and turns will be approximately halved. If this stripped coil is connected to the same DC voltage as used by the full coil, the current will double but the nimber of ampere-turns will remain constant. This stripped coil will thus have approximately the same effect on the magnetic core as the full coil but at twice the peak power and half the weight.

Figure 3 shows the tradeoff between coil weight and peak power for several magnet sizes. These curves do not include the effect of insulation on coil weight. Notice that for large peak power, the coil weight and powercan be traded one for one (i.e., doubling the coil weight reqires half the power), but small peak power results in a considerable weight penalty because of coil build-up effects.

Insulation weight will range from 5% of coil weight for high power coils to 15% for low power coils.
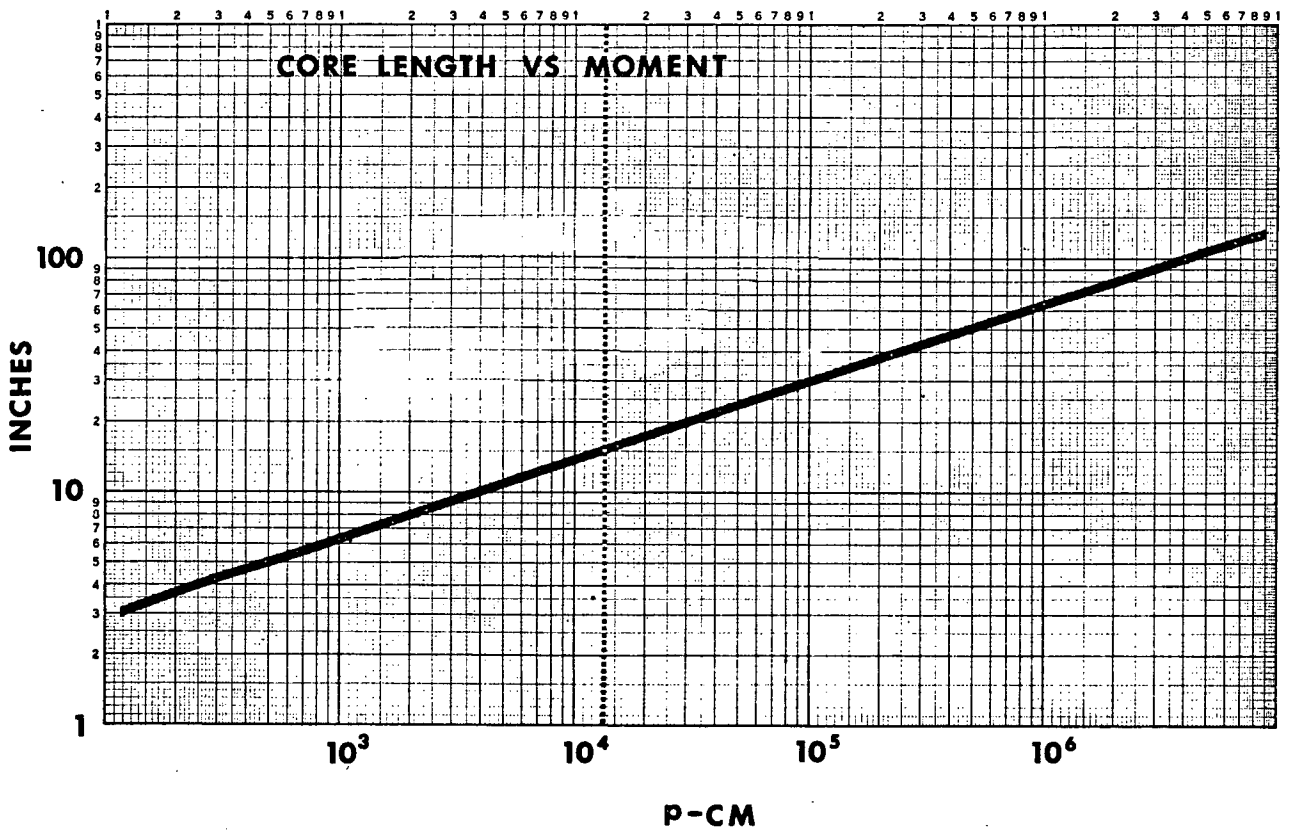
Figure 1. Length of Core vs. Maximum Dipole Moment
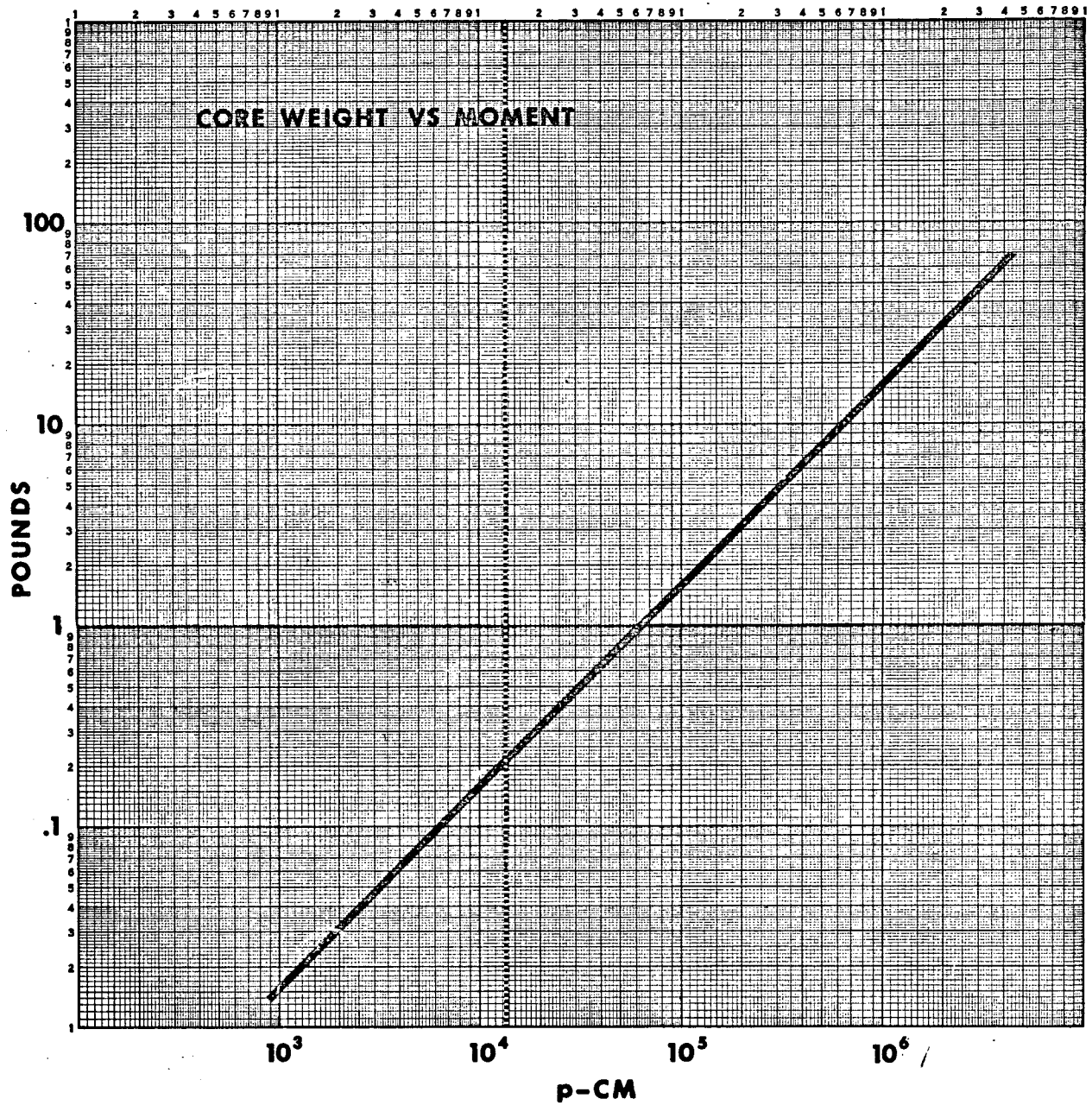
Figure 2.   Core Weight vs. Moment

β

6.3 Switching Energy

It is somewhat misleading to talk of power used by variable permanent magnets, as power is consumed only when the dipole moment is changed. A more meaningful concept is the energy used for each change. This energy goes to three sinks in the circuit, but must all come from the satellite power supply.

The first energy sink is the hysteresis of the magnet material. Any change in magnetic state requires energy proportional to the area of the B-H loop traversed. For well designed systems, this is the dominant energy sink. The second sink is I^2R loss in the coil.

The final sink is the charging circuit. The simplest and most efficient charging technique is to switch the coil directly to the spacecraft power supply line, in which case this loss is essentially zero. For most magnets, assuming there are no severe limits on the peak current available from the power supply, this is by far the preferred technique. It is possible to isolate the spacecraft power supply from these current transients by using either capacitors or rechargeable batteries, but this carries a definite system weight penalty.

Figure 4 can be used to estimate both the time and the total energy required to switch a magnet between its extreme states. This includes core and coil losses. From these curves it is evident that approximately 6×10^{-5} watt sec/pole cm is required to change the state of a magnet, more or less independent of magnet size. Thus a 10,000p-cm magnet can be switched between its extremes (M = 20,000 p-cm) at a cost of 1.2 watt sec of energy.

6.4 Example

The following example illustrates how to use these curves to determine the approximate magnet size and weight for a given set of requirements.

Maximum dipole moment required: 12,000 p-cm (12 amp m^2)

Power supply voltage: 25 V

Peak currnet allowable: .3 amps

From Figure 2, the weight of the core will be .2 lb.

The length of the magnet assembly as taken from Figure 3 is 15 inches.

To determine the coil weight, we use Figure 4 and estimate the position of the 12,000 p-cm curve. We estimate the coil weight for 7.5 watts and 12,000 p-cm as 0.4 lb.

The magnetizing time for the magnet can be estimated from Figure 5. For a 12,000 p-cm magnet at 7.5 watts this time is approximately .14 seconds.

The estimated parameters are tabulated below for comparison with an actual magnet assembly.

	Estimated	Actual
Maximum Dipole Moment	12,000 p-cm	12,300 p-cm
Core weight	.2 lb	.22 lb.
Coil length	15 inch	15 inch
Coil weight	0.4 lb	0.48 lb
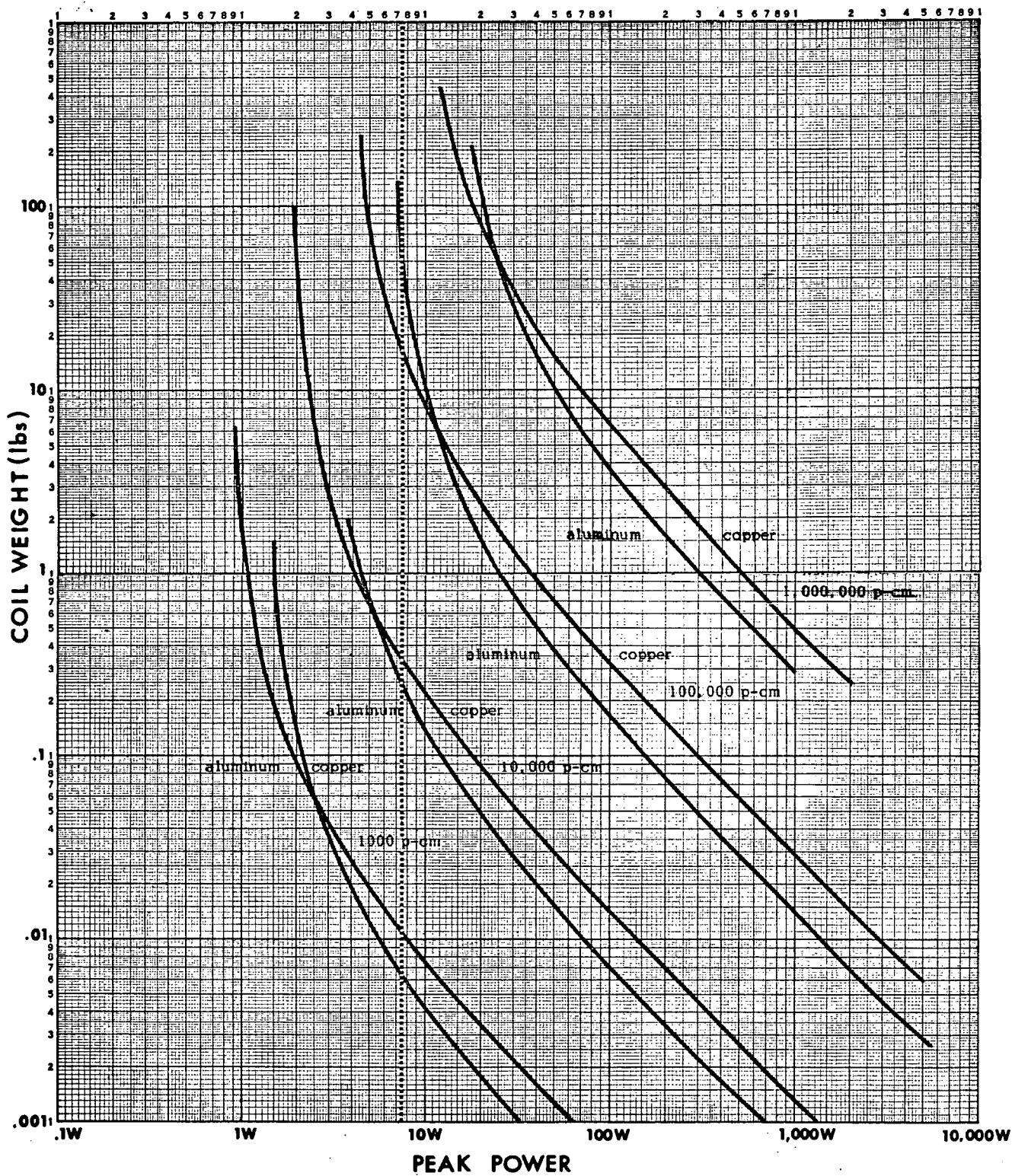
Figure 3.   Coil Weight vs. Power

Figure 4. Magnetizing Time vs. Peak Power
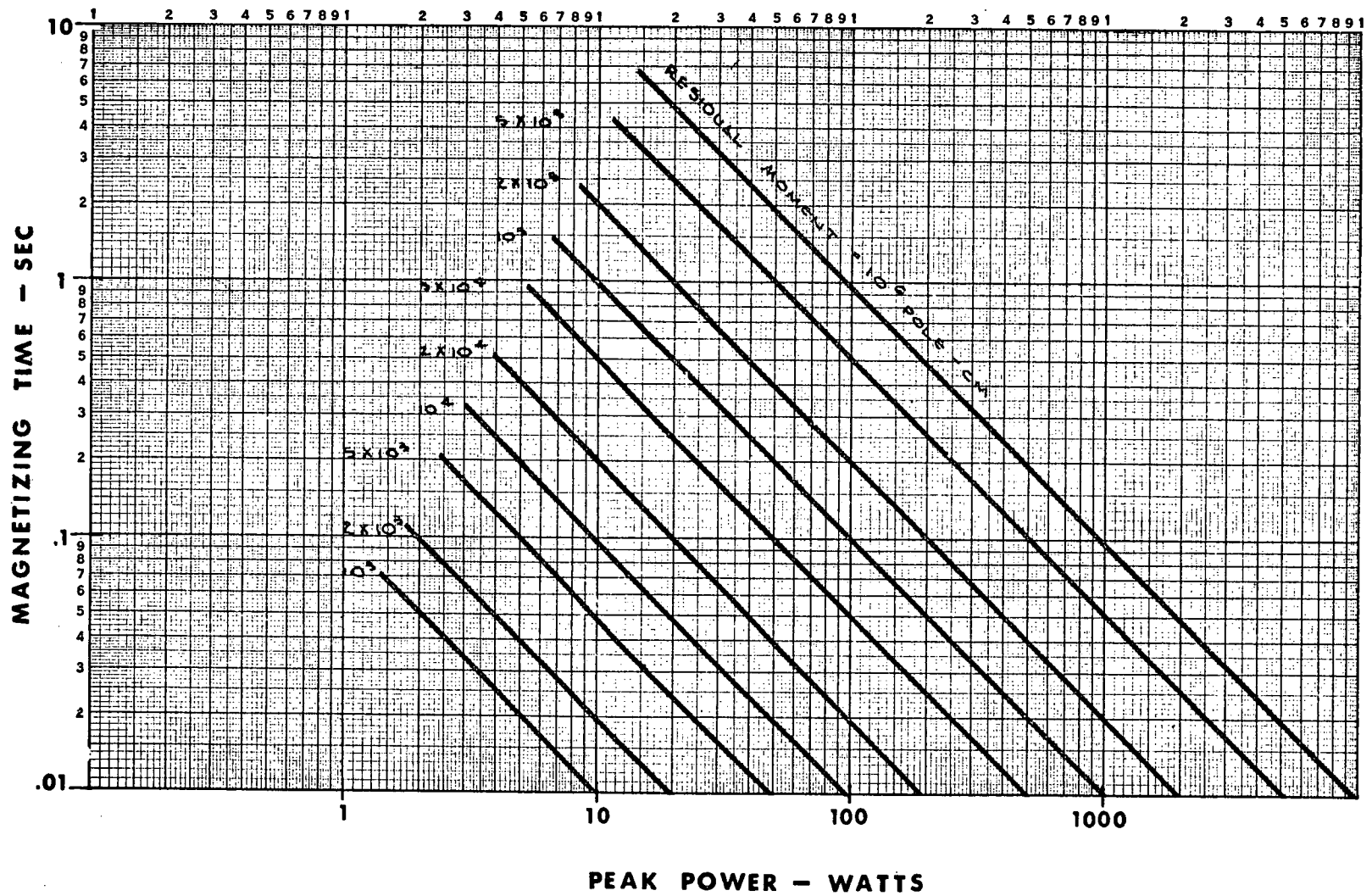
7.0 Description of Final Magnet Design and Performance

7.1 Introduction

This section presents test results and construction details for an elliptical chargeable permanent magnet. The test magnet is in the form of a prolate ellipsoid with integral magnetizing coil.

7.2 Sensing

One of the most difficult design problems is the implementation of the sensor for determining the state of the magnet. Such a sensor is desirable in balancing applications for telemetry purposes, and is essential in control applications because there is no way to properly control the state of the magnet without feedback from such a sensor.

The most convenient sensing device is the Hall sensor, which may be attached directly to the magnet. The primary difficulty arises from the fact that the magnetic state in the vicinity of the sensor does not necessarily represent that of the entire magnet. The accuracy of the sensed moment is therefore dependent on magnet history. The optimum configuration of the many that were investigated was for the sensor to be mounted near the end of an elliptical core, with a soft iron pole piece outboard of the sensor. This configuration is described in the drawings, and gives the sensing accuracy described in the accompanying curves.

7.3 Test Results

The magnet core is a 14 inch long by 0.34 inch diameter prolate ellipsoid and the weight of the magnet core and magnetizing coil is 0.575 pounds (262 grams).

The maximum residual magnetism for this magnet is 13,400 pole-cm and accuracy of the telemetry is $\pm$200 pole-cm ($\pm$1.5%). The accuracy increases when the magnet is utilized to less than its maximum moment capability – but a large decrease in moment is accompanied by a small increase in accuracy. These effects can be seen in the sensed moment vs actual moment curves of figures 1 and 2.
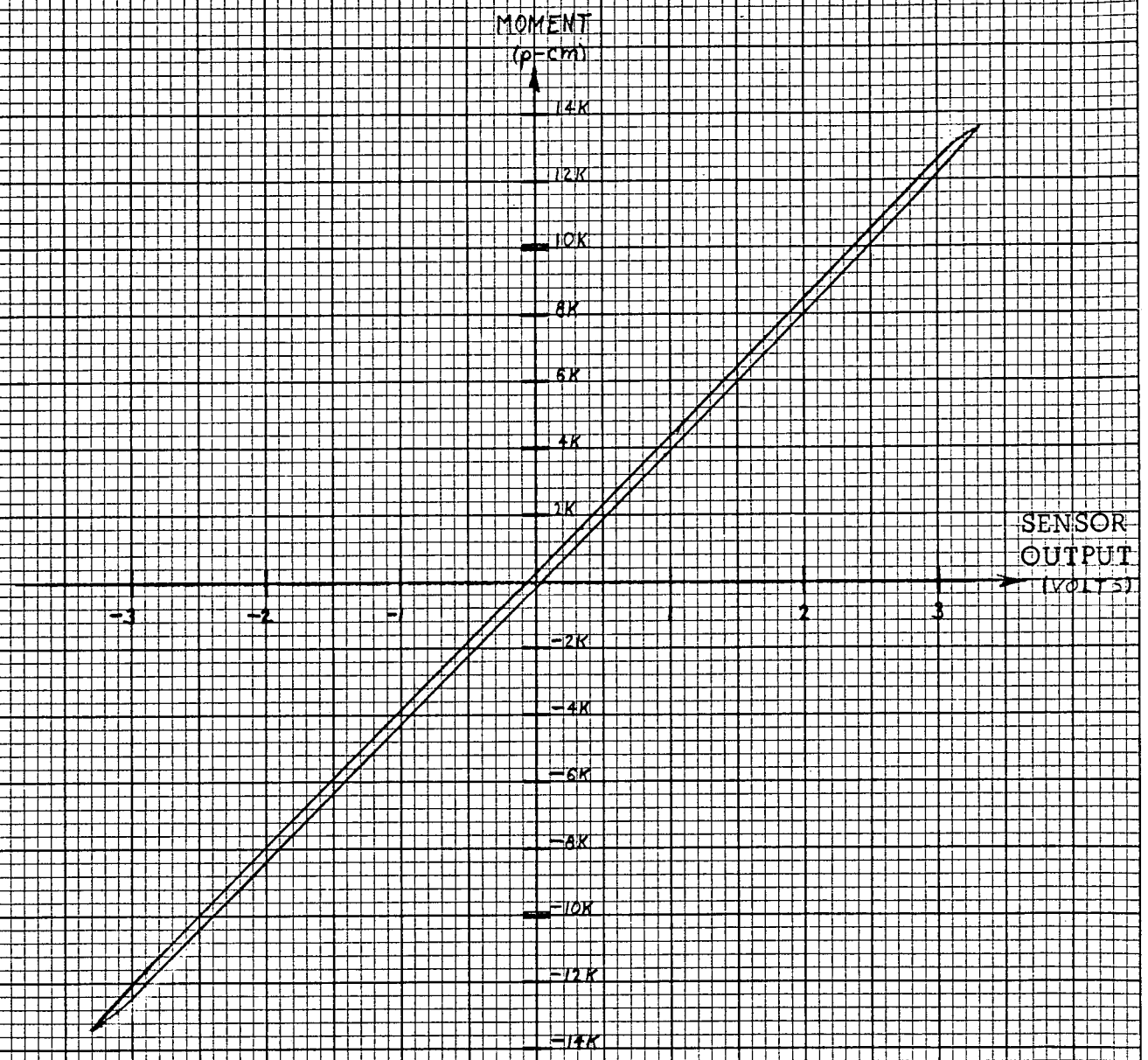
Figure 7-1 MOMENT vs. SENSOR OUTPUT

Elliptical Magnet s/n 15031

7.4 Magnet Construction

The test magnet was fabricated in the same major steps as are all ITHACO magnets. These steps are: fabricating the magnetic core which includes bonding laminations into a stack and grinding to final form; magnet cap assembly; winding the coil; potting and housing the complete magnet.

7.4.1 Magnet Core

Construction of magnetic cores begins by cutting Remendur 38 core material to rough size from 0.02 inch strip stock. The rough cut strips are heat treated at 1100°F in an inert atmosphere. After a two hour bake the strips are cooled at 100°F per hour to 900°F and then removed from the oven to cool in ambient air.

The heat treated strips are next bonded together with epoxy to form a rectangular bar and then ground to final shape. This magnet was ground to an elliptical shape with one end cut off square as shown in figure 3a.

7.4.2 Magnet Cap

The magnet caps are 3 piece assemblies as shown in sketch SK00224. Part #1 in this sketch is used as a partial coil form on both ends of the magnet. Figure 3b shows parts #1 bonded to the magnet core.

One end of the magnet core is ground flat to provide a flat face for the Hall sensor. The sensor is a small rectangular plate and is bonded to SK00224 part #3. Part #3 is then slid into part #2 with the sensor facing out, the sensor leads are placed in the wire channel and this assembly is slid into part #1. The sensor is then sandwiched between part #3 and the flat end of the magnet core. The final assembly step for the magnet cap is securing the plug part #2 in place with set screws. The end caps and core are now ready for winding the coil.

7.4.3 Magnetizing Coil

This magnetizing coil is a conventional solenoid consisting of 3703 turns of #26 AWG copper wire in 5 layers. After winding, the coil was coated with PC22 polyurethane and cured. The magnet assembly is shown in figure 3c before conformal coating.

Magnet initially AC demagnetized,
then cycled to 3k p-cm, -3k p-cm,
9k, -9k, 9k, -13k, 13k and finally
to -13k p-cm.

Figure 7-2 MOMENT vs. SENSOR OUTPUT

Elliptical Magnet s/n 15031

At this point the magnet assembly is functionally complete. The remaining operations are inserting the magnet into its housing, potting the assembly and painting the housing.

7.4.4 Potting and Housing

The housing selected for the test magnet was a 15 inch length of fiberglass tubing with an inside diameter of 5/8 inch. The space between the magnetizing coil and the tube was filled with PR1524 polyurethane.

This complete magnet assembly is 15 inches long x 3/4 inch diameter and weighs 1.0 pounds including a 6 ft. cable.

7.5 Test Method

The magnet was tested by placing the magnet on a wood table and sensing the remote field of the magnet with a flux gate magnetometer. The magnetometer probe was positioned parallel to the long axis of the magnet at a spacing of 45.5cm. At this spacing the calculated scale factor is 10 milligauss per 1000 pole-cm.

The magnetizing coil was energized for this test by a 1400uf bank of capacitors in the MMCA Test Panel. The MMCA Test Panel also has a polarity reversing switch and a selector switch which connects the capacitor bank to either an external power supply or to the magnetizing coil.

An amplifier was used to increase the output voltage of the Hall sensor to a more convenient level. As shown in figure 4, this consists of a differential amplifier with a gain of 50, which provides an offset adjustment control and also includes a bias network for the Hall sensor.

The entire test was conducted with all equipment on two wood tables. One table was used for the magnet under test and the magnetometer probe while all other equipment sat on the second wood table. This was done to minimize magnetic interference due to ferrous parts in the test equipment.
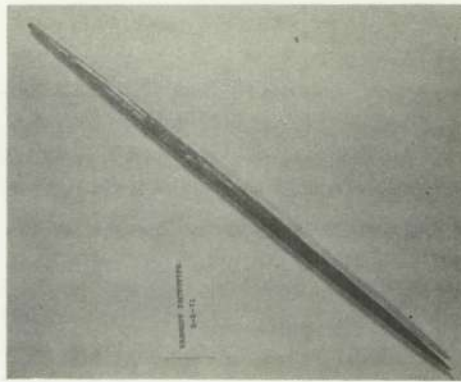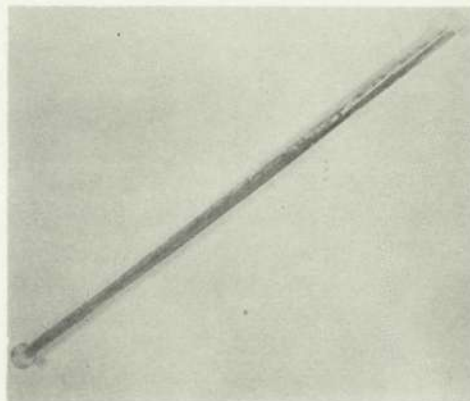
Figure 7-3a
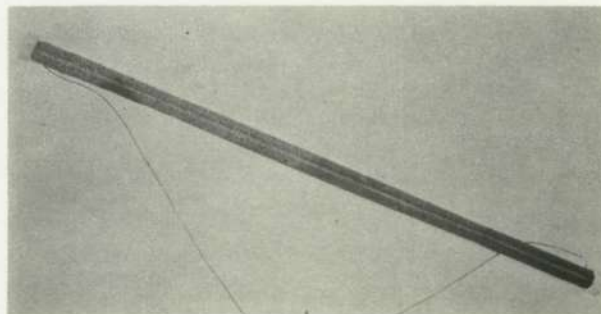
Figure 7-3b

Figure 7-3c

Figure 7-3 Magnet Construction

7-6

Figure 7-4   Test Set-Up

SECTION B-B

.125
.312

.G25

4-40 THRU
2 HOLES
180° APART

.250

.280

1- END CAP
MAT'L:
POLYCARBONATE

STA
.48

STA
1.00

STA
1.8

STA
2.1

.234 (5/A) DIA
.50

.159 (#3) DIA
1.10

STA
.33

.166 (#19) DIA
1.60

.136 (#29) DIA
1.95

STA
.02

.089 (#43) DIA
2.27

2.58

STA-2.6

.100

.264

REAM W/ REAMER SHAPED
BY MAGNET CAM

STA .33

STA 2.6

.062 WIDE X
.030 DP
WIRE CHANNEL

.040 'V' WIDE
(GROOVE)

.157 (#22) DIA

.249 DIA

2- PLUG
MAT'L:
POLYCARBONATE

.062

.280

.835

SECTION A-A

3- PLUG MAGNET
MAT'L:
CRS

.156 DIA

.13
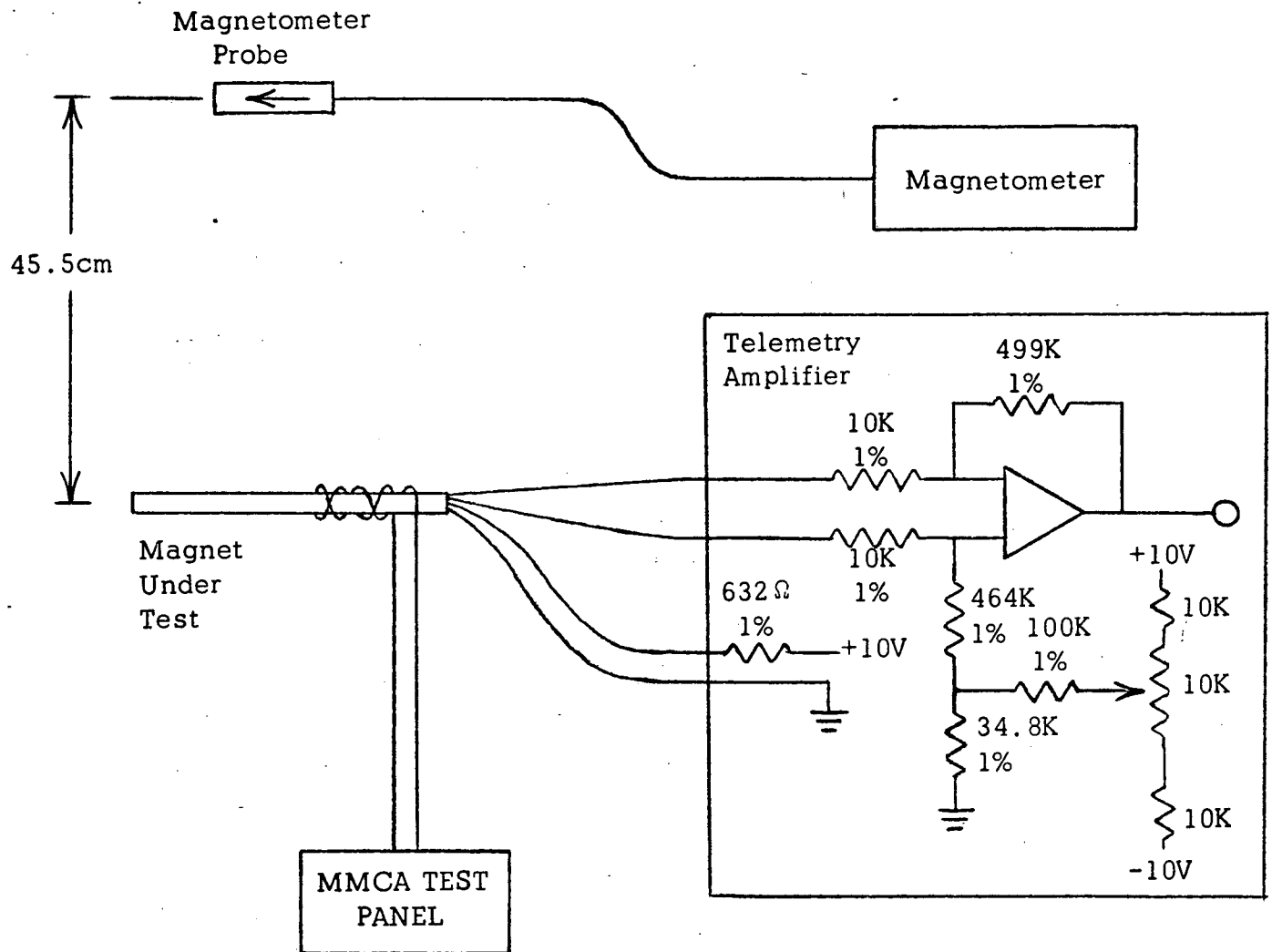
II MAGNETIC ACQUISITION SYSTEM IMPLEMENTATION

1.0 Introduction

2.0 Description of Hardware

3.0 Magnetometer Noise Measurements

Appendix - A Magnetic Control System for Attitude Acquisition

II MAGNETIC ACQUISITION SYSTEM IMPLEMENTATION

1.0 Introduction

An automatic attitude acquisition system has been developed under contract NAS5-21649. A description of this system and its performance is given in report no. 90345, which is included as an appendix to this section of this report.

The development of the hardware for this system was done under contract NAS5-21170, and is reported here.

2.0 Description of hardware

The hardware required for the magnetic acquisition system includes a 3 axis magnetometer, rate circuits, and 3 magnets. The magnetometer developed under this program is similar to other magnetometers made by Schonstedt Instrument Company, but repackaged for the applications. On the following pages will be found:

a) A preliminary specification for a three-axis magnetometer.

b) A Schonstedt control drawing for the magnetometer built to this specification.

c) Test data on the engineering model magnetometer.

d) A photograph of the magnetometer probe.

PRELIMINARY

SPECIFICATION FOR

A THREE-AXIS MAGNETOMETER

PREPARED BY:

ITHACO, Inc.
735 W. Clinton Street
Ithaca, New York 14850

SPECIFICATION FOR

A THREE-AXIS MAGNETOMETER

Approved by:_____   Date:_____
　　　　　　　J. S. Levy
　　　　　　　Project Engineer

_____   Date:_____

R. R. Fleming
R&QA Engineer

_____   Date:_____

D. Sonnabend
Program Manager

_____   Date:_____

R. Z. Fowler
Vice President

TABLE OF CONTENTS

1.0   SCOPE

This specification establishes the interface requirements and
performance of a set of three magnetometers for use in the SATS
attitude control system.   In addition, this document references
documents which pertain to the quality of workmanship and parts
to be used.

2.0   APPLICABLE DOCUMENTS

The following documents form a part of this specification to
the extent specified herein.   In the event of a conflict between
documents referenced here and the detailed content of this speci-
fication, the latter shall take precedence.

2.1   NASA Publications

| NHB 5300.4 (3A) May 1968 | Requirements for Soldered Electrical Connections |
| PPL-11 Revision A July 1971 | GSFC Preferred Parts List |

2.2   ITHACO Drawings

| D41017 | Frame, Electronics Univ. |
| C88135 | Printed Circuit Card, Control Drawing |

3.0   SPECIFICATIONS

3.1   Electrical Characteristics

The vendor shall supply three output voltages.   Each
voltage shall be proportional to one of three orthogonal
components of the ambient magnetic field.   A signal common
shall also be supplied. The following specifications apply
over the operating temperature range 0-50°C.

3.1.1   Operating Sources

The magnetometer shall be capable of operating
from externally supplied +10VDC and -10VDC ±1V.

3.1.2   Sensitivity

The three components of the magnetic field shall
be related to the respective output voltages by
the factor 4VDC/Oersted ±2%.

3.1.3   Bi-polarity

Reversal of the probe direction will cause the
output voltage to reverse sign.

3.1.4 Minimum Saturation Levels

The magnetometer shall not saturate when subjected to a magnetic intensity of 1 Oersted along the sensitive axis of the probe.

3.1.5 Null

In the absence of a magnetic field, the output voltage shall not exceed 20mVDC (at +25°C).

3.1.6 Linearity

The magnetometer output shall be linear to within 0.5% of full scale. Full scale shall be defined as 0.80 Oersted.

3.1.7 Ripple Voltage

The maximum allowable ripple voltage at the output of the magnetometer is 20mV peak-to-peak.

3.1.8 Temperature Stability

The change in null output voltage over the ambient range 0°C to 50°C shall be no greater than 30mV.

3.1.9 Noise

After processing by a test circuit, the output noise voltage shall be no greater than 32mV peak-to-peak. The figure is a schematic of the test circuit.

3.1.10 Crosstalk and Sensitivity to Crossfield

With all three magnetometers energized, apply the magnetic field along the mechanical axis of one of the magnetometers. The resultant voltage in each of the other two aces shall be equivalent to not more than 1 degree in misalignment. The null voltage shall be subtracted before performing the calculations.

3.1.11 Power Consumption

Average power shall not exceed 750mV.

3.2 Environmental Tests

The launch and orbit conditions to which flight models of this design will be subjected shall be considered.

3.2.1   The satisfactory operation must be maintained over the temperature range from -10°C to +60°C. Some degradation in analog relationships is permissable at the temperature extremes, but all operations must function properly.

3.2.2   The magnetometer shall be capable of meeting the requirements of Section 3.1 after being vibrated at the following levels with power applied.

<u>Sinusoidal Vibration</u>

Frequency Range (Hz)	Amplitude - "g" 0-to-Peak Thrust Axis
5-20	0.5" D.A.D.
20-30	10g
30-85	15g
85-110	37g
110-2000	15g

	Transverse Axis
5-21	0.5" D.A.D.
21-30	16g
30-2000	15g

<u>Random Vibration</u>

Direction	Frequency Band (Hz)	Spectral Density (g^2/Hz)	g-RMS
Thrust Axis	20-2000	0.2	20
Transverse Axis	20-2000	0.2	20

The duration of the test shall be 4 minutes in each direction -- 12 minutes total.

3.3   <u>Operability</u>

3.3.1   Maintainability

The equipment shall be designed so that maintenance shall not be required during the specified operating and storage period.

3.3.2 Shelf Life

The equipment shall be capable of meeting the requirements of Section 3.1 after a two-year storage period, under the following environments:

Temperature   5 to 45°C
Relative Humidity   50% or less

3.3.3 Operating Life and Environment

The magnetometer shall be designed to function in the free space environment for a period of three years.

3.4 Circuit Packaging

The magnetometer electronics shall be packaged subject to the constraints imposed by the drawings listed in Sec. 2.2.

3.5 Probe Packaging

This section will be added later.

3.6 Hand Soldering

The provisions of NHB 5300.4(3A) shall apply to all hand soldering operations.

3.7 Selection of Components

Parts should be chosen from the GSFC Preferred Parts List (PPL-11, Revision A). Parts which are not on the Preferred Parts List shall be approved by ITHACO.

4.0 PREPARATION FOR DELIVERY

The subcontractor shall package the magnetometer in such a way as to prevent damage or deterioration during shipment. Packaging shall be subject to review and approval by ITHACO. All shipping containers are to be plainly marked with ITHACO purchase order number, name, quantity, serial number and ITHACO specification number.
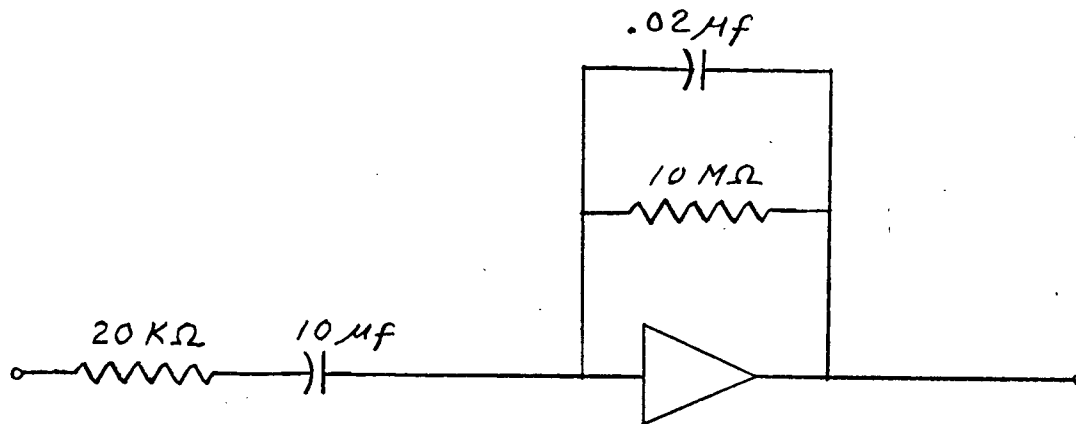
TEST CIRCUIT FOR SECTION 3.1.9

(Circuit supplied by ITHACO)

Magnetometer Probe

2-1

REV LET.	REVISION	APPROVED	DATE
A	15 FT. WRG 6 EV.		6-23-72

J1 REF.

J2 REF.

6.060 REF

.750 REF

.938 REF

6.250 REF

6.625 REF

VIEW WITH COVERS REMOVED
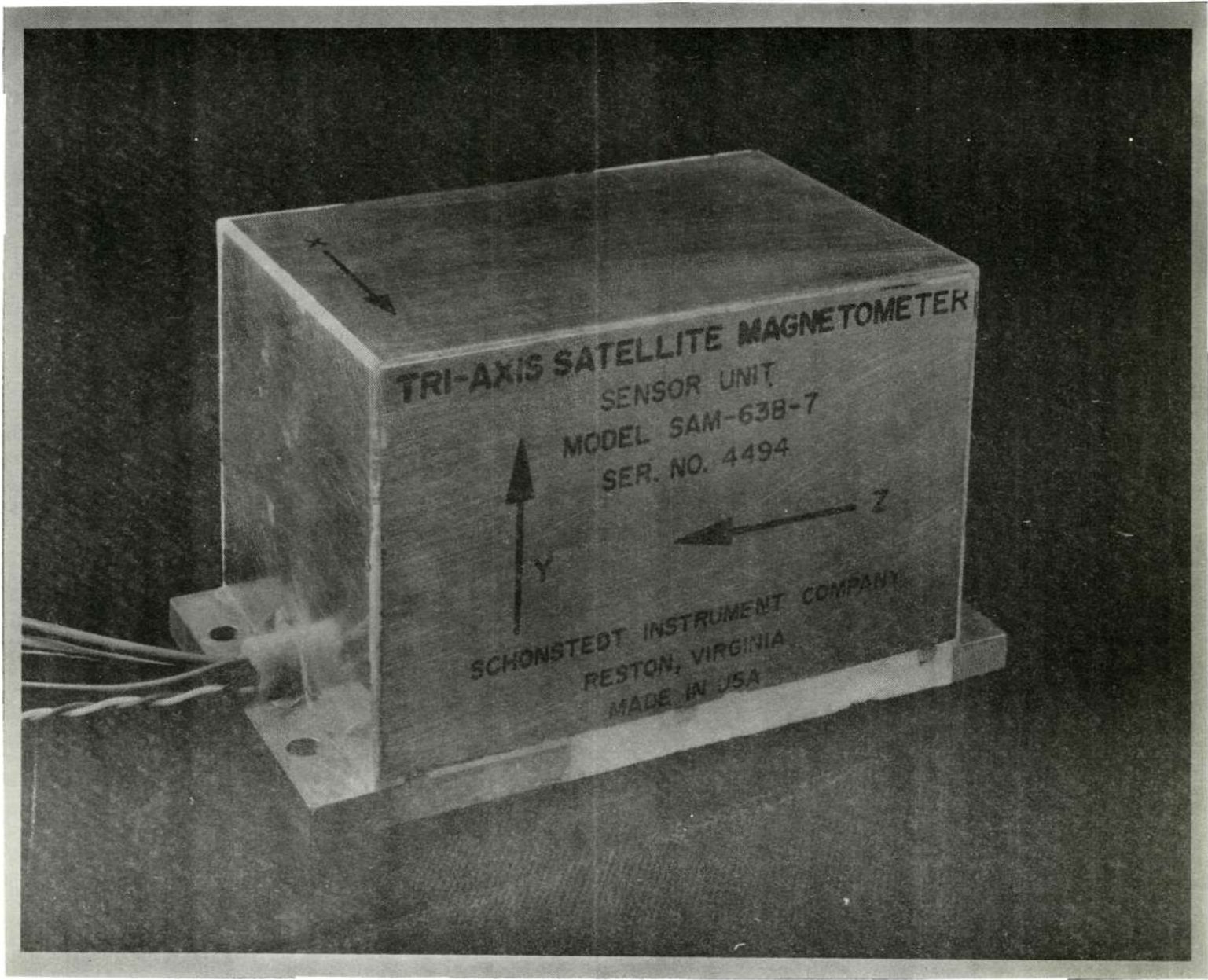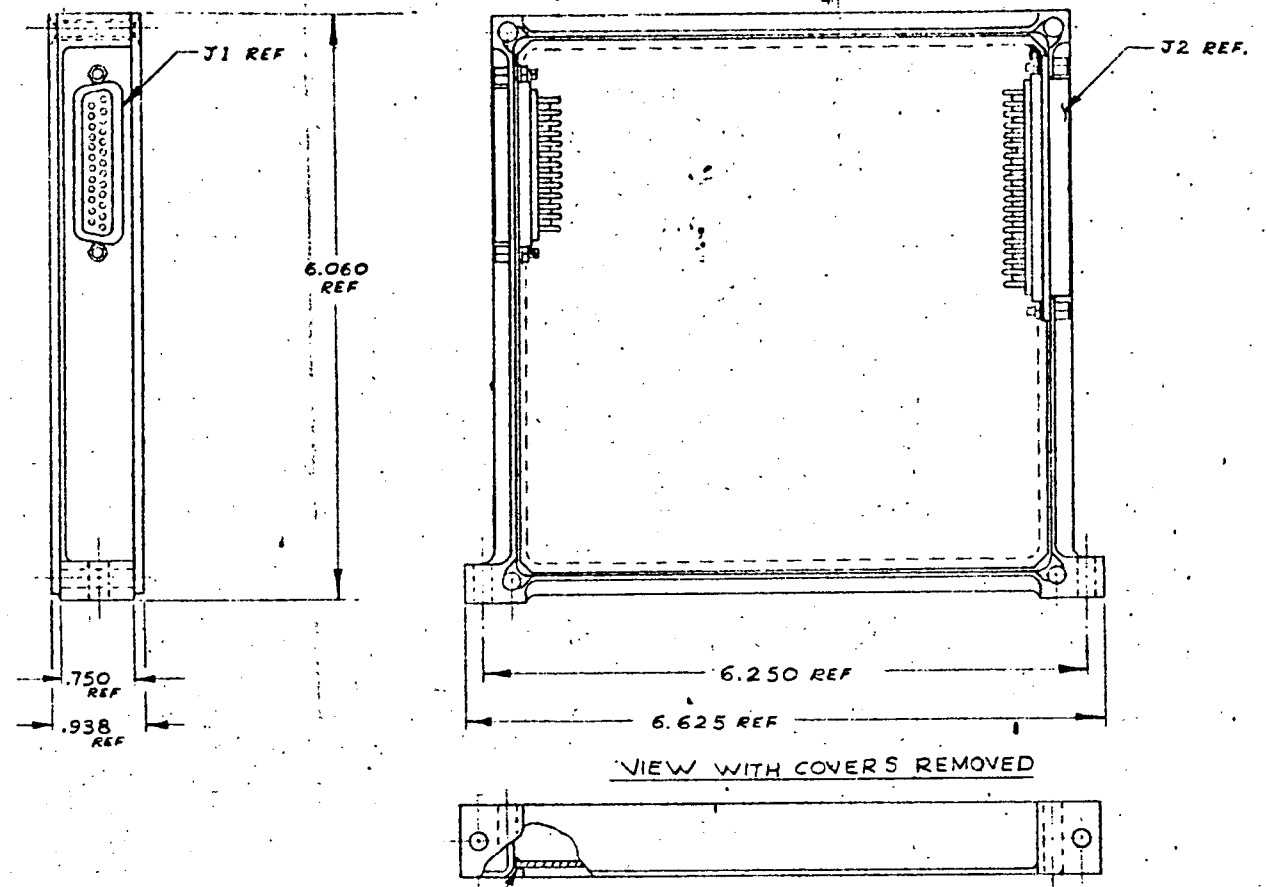
ADHESIVE
ECCOBOND EC-55
#9 CATALYST

ALIGNMENT CORNER

ALIGNMENT CORNER

2.765 MAX

1.375

*1.390

* THESE DIMENSIONS ARE TO THE
CENTER OF THE SENSING
ELEMENTS

ARROWS INDICATE DIRECTION
OF MAGNETIC FIELD FOR
OUTPUT SIGNALS GREATER
THAN THE BIAS LEVEL

4.687
3.968 MAX
.359 MIN
4.313
.187
1.500
2.750 BASE SIZE
2.781 MAX
.625
.687
.201 DIA.
4 HOLES
1.281
2.937

TRI-AXIS SATELLITE MAGNETOMETER
SENSOR UNIT
MODEL SAM-63B-7
SER. NO.
SCHONSTEDT INSTRUMENT COMPANY
RESTON, VIRGINIA
MADE IN USA

P1 REF.

15 FEET
± 6 INCHS

.250

CANNON CONNECTORS	
P1	DBMM-25S-
J1	DBMM-25P-F
J2	DCMM-37P-F
P2	DCMM-37S-

TO SENSOR
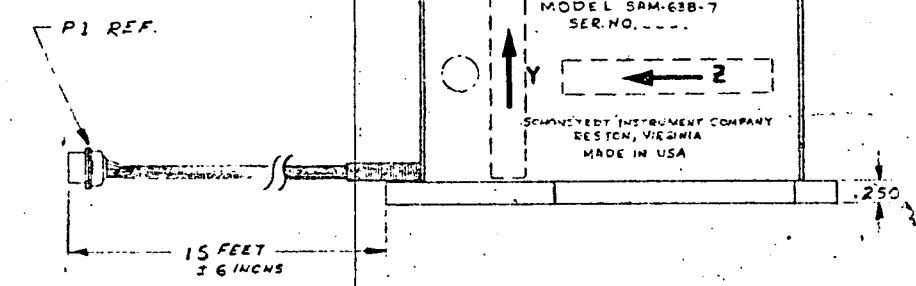
ELECTRONIC UNIT

POWER & TELE.

		P1	J1				J2	P2			
EXC	BLU	1	1	EXC	CHASSIS GND	19	19	WHT	CHASSIS GND	
EXC	WHT	3	3	EXC		37	37	WHT		
X SIG	BLU-BLU	10	10	X SIG IN	0V	18	18	BLK	SIG 0V	
		23	23	0V		36	36	BLK		
Y SIG	ORA-ORA	8	8	Y SIG IN	SIG OUT	17	17	BLU	Z SIG	
		21	21	0V		35	35	BLU		
Z SIG	BLK-BLK	6	6	Z SIG IN		16	16	YEL	Y SIG	
		19	19	0V		34	34	YEL		
						15	15	VIO	X SIG	
						33	33	VIO		
END	GRN	25	25	CHASSIS GND	-10V	14	14	ORA	-10V	
			13			32	32	ORA		
					+10V	13	13	RED	+10V	
						31	31	RED		

6 ft. 6 in.
TEST CABLE
NOT FURNISHED WITH INSTRUMENT

REV
A

401284

SCHONSTEDT INSTRUMENT COMPANY

SAM-63B-7   Ithaco P/N C188168   Serial No. 4494

TEMP. 25°C

Input Voltage: +10 ±0.1 VDC
                 −10 ±1.0 VDC

FIELD	OUTPUT SIGNAL VDC			LIMITS, VDC	
Moe	X	Y	Z	Min.	Max.
+1000	3.992	3.995	3.982	+4.0 Nominal	
+800	3.197	3.195	3.184	+3.136*	+3.264*
+700	2.797	2.796	2.786	+2.744*	+2.856*
+600	2.398	2.396	2.388	+2.352*	+2.448*
+500	1.999	1.997	1.990	+1.960*	+2.040*
+400	1.599	1.598	1.591	+1.568*	+1.632*
+300	1.199	1.198	1.193	+1.176*	+1.224*
+200	0.800	0.799	0.795	+0.784*	+0.816*
+100	0.400	0.399	0.397	+0.392*	+0.408*
ZERO	0.003	0.000	−0.003	−0.020	+0.020
−100	−0.400	−0.401	−0.401	−0.408*	−0.392*
−200	−0.798	−0.800	−0.800	−0.816*	−0.784*
−300	−1.198	−1.199	−1.198	−1.224*	−1.176*
−400	−1.597	−1.599	−1.596	−1.632*	−1.568*
−500	−1.996	−1.998	−1.994	−2.040*	−1.960*
−600	−2.395	−2.397	−2.393	−2.448*	−2.352*
−700	−2.795	−2.797	−2.791	−2.856*	−2.774*
−800	−3.194	−3.196	−3.189	−3.264*	−3.136*
−1000	−3.994	−3.995	−3.986	−4.0 Nominal	

TEMPERATURE TEST

TEST No._____   PS 100292

SCHONSTEDT INSTRUMENT COMPANY

SAM-63B-7   Ithaco P/N C188168   Serial No. _4494_

TEMP. _50°C_

Input Voltage: +10 ±0.1 VDC
              -10 ±1.0 VDC

FIELD	OUTPUT SIGNAL VDC			LIMITS, VDC	
Moe	X	Y	Z	Min.	Max.
+1000	3.991	3.986	3.978	+4.0 Nominal	
+800	3.192	3.198	3.182	+3.136*	+3.264*
+700	2.793	2.790	2.784	+2.744*	+2.856*
+600	2.395	2.392	2.386	+2.352*	+2.448*
+500	1.996	1.993	1.988	+1.960*	+2.040*
+400	1.597	1.594	1.591	+1.568*	+1.632*
+300	1.198	1.195	1.193	+1.176*	+1.224*
+200	0.799	0.797	0.795	+0.784*	+0.816*
+100	0.401	0.398	0.397	+0.392*	+0.408*
ZERO	0.001	-0.001	-0.001	-0.020	+0.020
-100	-0.397	-0.401	-0.400	-0.408*	-0.392*
-200	-0.796	-0.799	-0.797	-0.816*	-0.784*
-300	-1.195	-1.197	-1.195	-1.224*	-1.176*
-400	-1.594	-1.596	-1.593	-1.632*	-1.568*
-500	-1.992	-1.995	-1.991	-2.040*	-1.960*
-600	-2.392	-2.394	-2.388	-2.448*	-2.352*
-700	-2.791	-2.792	-2.786	-2.856*	-2.774*
-800	-3.190	-3.191	-3.184	-3.264*	-3.136*
-1000	-3.988	-3.988	-3.980	-4.0 Nominal	

SCHONSTEDT INSTRUMENT COMPANY

SAM-63B-7   Ithaco P/N C188168   Serial No. _4494_

TEMP. _0°C_   Input Voltage: +10 ±0.1 VDC
                                     -10 ±1.0 VDC

FIELD	OUTPUT SIGNAL VDC			LIMITS, VDC	
Moe	X	Y	Z	Min.	Max.
+1000	3.992	4.014	3.974	+4.0 Nominal	
+800	3.195	3.211	3.176	+3.136*	+3.264*
+700	2.797	2.812	2.778	+2.744*	+2.856*
+600	2.398	2.413	2.379	+2.352*	+2.448*
+500	1.999	2.014	1.981	+1.960*	+2.040*
+400	1.600	1.615	1.582	+1.568*	+1.632*
+300	1.201	1.215	1.183	+1.176*	+1.224*
+200	0.801	0.811	0.784	+0.784*	+0.816*
+100	0.402	0.413	0.385	+0.392*	+0.408*
ZERO	0.003	0.013	-0.005	-0.020	+0.020
-100	-0.397	-0.387	-0.414	-0.408*	-0.392*
-200	-0.795	-0.787	-0.812	-0.816*	-0.784*
-300	-1.195	-1.186	-1.211	-1.224*	-1.176*
-400	-1.594	-1.585	-1.609	-1.632*	-1.568*
-500	-1.993	-1.984	-2.008	-2.040*	-1.960*
-600	-2.392	-2.383	-2.407	-2.448*	-2.352*
-700	-2.791	-2.782	-2.806	-2.856*	-2.774*
-800	-3.190	-3.180	-3.204	-3.264*	-3.136*
-1000	-3.988	-3.977	-3.978	-4.0 Nomin ⸱	

3.0 Magnetometer noise measurements

One concern in the designs of this magnetic acquisition system was whether or not available magnetometers have sufficient sensitivity to be able to detect the very low fixed rates required. Three of the Schonstedt RAM5C-NB Magnetometers were purchased and evaluated, using circuitry similar to the differentiators that will be used in the final hardware. The results of these tests are given in the following report no. 90293.

MAGNETOMETER NOISE MEASUREMENTS

PURPOSE OF THE STUDY

This study is part of an attempt to detect apparent changes in the earth magnetic field from a satellite during initial attitude acquisition. A system designed to accomplish this (Figure 1) consists of the Schonstedt RAM-5C-NB Magnetometer followed by a differentiator circuit. The output of the differentiator would be fed to a threshold detector of an attitude control system. This report evaluates the noise type and levels present in the magnetometer-differentiator combination.

SENSOR SYSTEM
FIGURE 1

GENERAL OPERATION OF THE MAGNETOMETER-DIFFERENTIATOR

The proposed magnetic aspect system (Figure 1) is composed of a probe and associated electronics (Magnetic Aspect Sensor) purchased from Schonstedt, and a differentiator circuit. The sensor provides a "d.c." output voltage proportional to the component of the magnetic field coaxial with the probe. The differentiator circuit not only differentiates, but provides attenuation over a range of frequencies outside of the range of vehicle rotation frequencies. Primarily, two circuits were used during the study, one with a corner frequency W_c of 1 radian per second and the other with a corner frequency of 5 radians per second. The approximate equivalent circuits are shown in . Figure 2.

DIFFERENTIATOR WITH $W_c = 1$

FIGURE 2a

DIFFERENTIATOR WITH $W_c = 5$

FIGURE 2b

MODEL

$$Y(j\omega) = \frac{100\omega}{1+\omega^2}$$

10 MΩ

0.1 μf

10 μf

100 KΩ

10 DEGREE PHASE
SHIFT FROM IDEAL
DIFFERENTIATOR

40 db

20 db

0 db

-20 db

-40 db

DATE

-60 db

TRANSFER FUNCTION FOR
DIFFERENTIATING THE APPARENT
EARTH MAGNETC FIELD

ω = .001   ω = .01   ω = 0.1   ω = 1   ω = 10   ω = 100   ω = 1,000   ω = 10,000

ω RADIANS / SEC.

FIGURE 3.

MODEL

DATE

± 10 DEGREE PHASE
SHIFT FROM IDEAL
DIFFERENTIATOR

10 MΩ

.02 µf

20 KΩ 10 µf

$$Y(j\omega) = \frac{100\omega}{1 + \frac{3l_2}{25}}$$

ω ORBIT
RATE

TRANSFER FUNCTION FOR DIFFERENTIATING
THE APPARENT EARTH MAGNETIC FIELD

ω RADIANS/SEC.

FIGURE 3b

The mathematical representation is

$$Y(jW) = \frac{-j100W}{(1+\frac{jW}{W_c})^2} \quad ; \; W \text{ is in radians per second}$$

Figures 3a and 3b are the Bode plots corresponding to the circuits of Figures 2a and 2b. Zero db is defined equivalent to unity gain.

TEST RESULTS

Figure 4 is the actual circuit configuration of the differentiator with 1 radian/sec. corner frequency. Separate FET's were used. The circuit with a corner frequency of 5 radians/sec. employed the 2N3922 dual FET's.

DIFFERENTIATOR SCHEMATIC
($W_c = 1$)

FIGURE 4

Three magnetometers SN3881, 3882, and 3883 were used at various times during the testing; but for consistency, 3883 was subjected to each test.

The outputs of the $W_c=1$ and $W_c=5$ circuits were measured when these circuits were used in combination with magnetometer 3883. The two tests were not run simultaneously. The magnetometer was centered radially in a wooden cylinder which in turn was centered radially in an iron pipe. The pipe was then closed. A magnet external to the pipe supplied the magnetic field. The magnet produced a 2.3 volt magnetometer output. The output of the differentiators were recorded using a Brush recorder.

The peak to peak values obtained are

     8mV for the W_c = 1 circuit,

     80mV for the W_c = 5 circuit.

Figures 5a and 5b are samples of the recordings from which the values were obtained.

To determine the noise generated by the differentiator circuit, the output was measured with the input shorted. Both circuits were placed in a metal box and the outputs simultaneously measured using the Brush recorder. The peak to peak output was half a millivolt for the W_c = 1 and 2 millivolts for the W_c = 5 circuit.

The magnetometers themselves do not have identical noise characteristics. Figure 6 is a Brush recording showing the noise output from the W_c = 5 circuit fed by each of the three magnetometers. Each of the magnetometers was tested in the pipe with the magnet supplying the same field in each case. A considerable difference between the noise levels of the magnetometers is apparent. Disturbances, some deliberately created, in the laboratory verify that the magnetometers and the differentiator were functioning during the measurements.

Two magnetometers, each in combination with a W_c = 1 differentiator were tested simultaneously to show that spikes which were being observed on the oscioolgraph are not internally generated. Oscillographs obtained during the night hours also tend to confirm that the largest spikes are not internally generated. The night graphs are almost without the large spikes which are common in the data taken during normal working hours.

Oscillographs were run for 1 hour with the W_c = 1 differentiator and then for 1 hour with the W_c = 5 differentiator. These recordings show that noise with a period on the order of minutes is negligible by comparison with the amplitude of the noise envelope.

Root-mean-square (RMS) measurements were obtained to indicate the general level of the noise as a function of W_c. The measurements were obtained graphically from the Brush oscillographs. Three circuits were used in turn with magnetometer 3883. The magnetometer was enclosed in the pipe and the field supplied by a magnet external to the pipe. All three differentiators were of the same mathematical form, all had K = 100, and all had the same input capacitor and output resistor. With those restrictions, the noise levels may be considered to be a function of W_c. The equivalent circuits for two of the differentiators are Figures 2a and 2b, the third (W_c = 0.5) is shown by Figure 7.

Vertical Scale: 1mV/small div.
Horizontal Scale: 5mm/sec.

Noise Output of Differentiator with W_c = 1 rad/sec. in
  Combination with Magnetometer SN 3883.

FIGURE 5A

Vertical Scale: 5mV/small div.
Horizontal Scale: 5mm/sec.

Noise Output of Differentiator with W_c = 5 rad/sec. in
  Combination with Magnetometer SN 3883.

FIGURE 5B

3882   3881   3883

Comparison of Noise Levels from Three Magnetometers.
  Output of Differentiator with W_c = 5 rad/sec.

FIGURE 6

DIFFERENTIATOR WITH $W_c = 0.5$
FIGURE 7

The results of the graphical analysis are given in Table 1.

W_c radian/sec.	Magnetometer 3883 Output Volts d.c.	Differentiator Output Noise Millivolts RMS
0.5	1.6	0.6
1.0	2.3	1.3
5.0	2.3	8.1

TABLE 1

To obtain the RMS voltage graphically, the instantaneous value of the noise voltage is measured at 1 second intervals. The square root of the mean of the square of these measurements is the RMS voltage.

MAGNETOMETER OUTPUT CHARACTERISTICS

Tests show that the d.c. output impedance of the magnetometer is 3.6K ohms. The 20K ohm resistance shown in Figure 2b actually includes the 3.6K ohm output impedance.

At a frequency of W_c = 0.5 radian/sec., the impedance of the 10μ farad differentiator input capacitor is 200K ohms, so there is neglibible loading from d.c. to W = 0.5 radian/sec.

Discussions with the vendor indicate that no electrical isolation problems exist in connection with the output of the magnetometers.

CONCLUSIONS

To get some idea of the feasibility of using the particular magnetometer-differentiator combination studied, some assumptions of physical conditions will be made.

For purposes of this study, a satellite will be assumed in a 500 mile high circular orbit. The RAM-5C magnetometer has a sensitivity of 4 volts per oersted. Assuming a maximum sea level field strength of 580 milliguass in the orbit plane, and an inverse cube variation of field strength with orbit radius; a voltage of $\approx$ 1.6 volts maximum would be present at 500 miles.

The Bode plots, Figure 3, give the voltage gain of the corresponding differentiators as a function of the frequency of a sinusoidal input. For simplicity, assume the satellite to be rotating about an axis which is fixed relative to the earth. Assume we are interested in rotation frequencies from 0.5 radians/sec. down to the orbit rate, and assume the orbit frequency is .001 radian/sec.

According to the Bode plots, a magnetometer signal at the orbit frequency would be attenuated by a minimum of 20db. For the 500 mile orbit, the peak differentiator output would be a maximum of ±160mV. If the angle between the magnetometer probe and the orbit plane were such as to reduce the field strength by a factor of 0.1, then the signal would be attenuated another 20db and the peak output of the differentiator would be ±16mV.

Suppose threshold detectors to be at the output of the differentiator. Based on the above discussion, the threshold levels might be set between ±16mV and ±160mV. Recall that the peak to peak noise values recorded on the oscillograph were

$\qquad$ 8mV for the $W_C = 1$ circuit,
$\qquad$ 80mV for the $W_C = 5$ circuit.

The oscillographs show that the noise level measured at the output of the differentiator is dependent on the d.c. output voltage of the magnetometer. The reason is not known.

The study does not conclusively define the limitations of this particular magnetometer-differentiator combination. However, based on the measurements, the prospects for this combination appear promising.

MAGNETOMETER NOISE MEASUREMENTS

Addendum to Report No. 90293, Aug. 27, 1971

Report No. 90293 gives the results of noise measurements for the Schonstedt RAM-5C-NB Magnetometer in combination with a differentiator circuit. As mentioned in that report, the noise level measured at the output of the differentiator is dependent on the d.c. output voltage of the magnetometer.

Oscillographs of the noise at the output of the magnetometer itself shows a dependence on the d.c. output voltage of the magnetometer. To obtain this more recent data, magnetometer 3883 was enclosed in the iron pipe and a constant magnetic field supplied by a magnet external to the pipe. The output of the magnetometer was coupled to the preamp of the Brush Recorder through a 10μf capacitor. Samples of these recordings are presented in the Figure.

A close look at the oscillographs show the noise level to be higher for a magnetometer output voltage of -2.4V d.c. than for an output voltage of +2.4V d.c. Since all of the data in Report No. 90293 was obtained for positive output voltages, the following question arises. If data were taken for negative and positive instead of just positive output voltages, might magnetometer 3881 be found to have greater p-p noise than 3883, instead of the other way around. To resolve this doubt, 3881 was tested in combination with the W_C = 5 differentiator; the magnetometer was enclosed in the pipe and the field supplied by a magnet external to the pipe.

The following table shows the p-p differentiator output noise as a function of magnetometer output voltage. The data is only approximate due to the difficulty of defining p-p noise. However, this data clearly shows that the noise levels are well below the 80mV p-p measured with 3883.

Differentiator Output Noise vs Magnetometer 3881 DC Output

Magnetometer Output Volts DC	W_C=5 Differentiator Noise Output Millivolts p-p
-2.4	25
-2.0	25
-1.0	14
0.0	9
+1.0	20
+2.0	19
+2.4	35

CONCLUSIONS

Noise levels vary from magnetometer to magnetometer. Noise levels also vary with field strength. Finally, and most important, if rates on the order of orbit rates are to be detected, the results of the tests with the $W_c = 5$ differentiator are inconclusive. Simulation will be needed to provide the answer. However, if a relay were installed to convert the $W_c = 5$ differentiator to a $W_c = 1$ differentiator during acquisition, detection at orbit rates should be feasible.

Jay Levy
ITHACO, Inc.
735 W. Clinton Street
Ithaca, New York 14850

vdc = +2.4

vdc = -2.4

vdc = +1.5

vdc = 0.0

vdc = +1.0

vdc = +0.5

horizontal scales:
    20mm/sec.
vertical scales:
    1mV/small division

Output Noise Voltage for Various DC Output Voltages for
Magnetometer S/N 3883

	+	−	
Current (Rm Temp)	44ma	25ma	

	X	Y	Z
Orthogonality	27mvp-p	4mvp-p	31mvp-p

	X		
Noise	7mvp-p	8mvp-p	8mvp-p
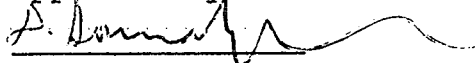
APPENDIX

A Magnetic Control System for Attitude Acquisition

A MAGNETIC CONTROL SYSTEM
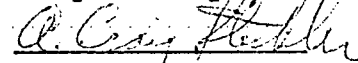
FOR ATTITUDE ACQUISITION

Approved by: Prepared by:

David Sonnabend   A. Craig Stickler
Program Manager   Systems Analyst

ABSTRACT

A Magnetic Control System for Attitude Acquisition

This report describes a spacecraft magnetic attitude acquisition system that is capable of automatically despinning a satellite from arbitrarily high rates around any axis, and provides terminal orientation that makes capture by conventional fine control attitude control systems routine. The system consists of a 3-axis magnetometer, a set of 3 orthogonal magnets, and appropriate control logic. No earth sensor is required.

Acquisition is treated in two phases. During the despin phase we are concerned with removing the tumbling motion of the satellite In this phase the performance of the system is unaffected by the presence or absence of a momentum wheel. Phase 2, that of orienting the spacecraft to the desired attitude, requires that a momentum bias wheel be present. In the terminal orientation, the axis of the momentum wheel is substantially perpendicular to the orbit plane (roll and yaw errors near zero) and the pitch rate of the satellite is at twice orbit rate.

This report describes the analysis and simulation that has been done in evaluating the performance of this system. A well-configured system will result in despin times of the order of 5 orbits per RPM for spacecraft in low earth orbits. Following despin, terminal orientation is achieved after another one to three orbits, depending on the capture range of the associated fine control system.

While this report does not describe the physical hardware, the system can be implemented inexpensively with weight less than 5 lbs. and power of about 3 watts.

CAST OF CHARACTERS
(In Order of Appearance)

$\bar{v}$	A vector (general)
v or $\lvert\bar{v}\rvert$	Magnitude of that vector
v_i	$\underline{i}$ th component of $\bar{v}$
$\bar{\bar{I}}$	Inertia tensor
I_i	Moment of inertia about $\underline{i}$ th coordinate axis
$\bar{\tau}$	Torque
$\bar{H}$	System total angular momentum
$\bar{\omega}$	Angular velocity
$\bar{h}$	System angular momentum contribution from wheels
T	Kinetic energy
$\bar{M}$	Magnetic dipole moment
$\bar{B}$	Magnetic flux density
ω_O	Orbital rate
$\bar{e}_n$	Unit vector along the orbit normal
K, α	Constants
Δt	Time intervals
$\bar{P}_O$	Initial spin momentum (not due to wheels)
T_{sys}	A parameter
Ω	Angular velocity
δx	A "small" change in x
I_{3e}	An equivalent moment of inertia about the 3 axis. Defined by Eq. (C7)
α_1, α_2	Defined by Eq. (E10) & (E11)
A	A matrix (page E-2 & E-3)
$\bar{x}(t)$	A vector whose components are $\{\delta\omega_1, \delta\omega_2, \delta B_1, \delta B_2\}$
s	The Laplace Operator
$\bar{X}(s)$	Laplace transform of $\bar{X}(t)$
L^{-1}	Inverse Laplace transform

TABLE OF CONTENTS

INTRODUCTION

In this report we will be concerned with the description and evaluation of a magnetic acquisition/attitude control system. The work described here has been performed during the last several months, principally by the author, with the assistance and guidance of Messrs. David Sonnabend, and Robert Fowler. This work was done under contract #NAS5-21649, at the behest of the Systems Analysis Branch, GSFC, Greenbelt, Maryland. The original concept of this system was suggested by Seymor Kant, Head, Systems Analysis Branch, GSFC.

The attitude control system described here is designed to "despin" a satellite and then orient it to any preferred Earth referenced attitude. This is done thru interaction with the Earth's magnetic field. An interesting feature of the system is that it requires no Earth sensor.

In this report we will discuss the control law for despinning the satellite and the mechanism thru which some preferred attitude is attained. Then we will discuss stability considerations as they apply to this system. Next, system performance is evaluated in the light of both analytical results and simulation results. We will be concerned with both despin and attitude acquisition performance. Finally, some recommendations for further study are included in case any parties might wish to continue the work discussed here.

Appendix B displays a listing of the FORTRAN program used in the digital simulation. Appendix C reviews some of the dynamic stability theory for dual spin satellites, for convenient reference, but does so in a different fashion than is usually employed. The results obtained are, of course, the same as usual. In Appendix D we analyze the equilibrium points of our system; the stability of these points is briefly treated in Appendix E, using some of the results of Appendix C.

The author wishes to make one further remark before continuing. The (differential) equations describing the behavior of the system discussed herein are nonlinear and otherwise intractable. This being the case, it is not possible to obtain explicit analytical solutions and recourse to simulation must be made. Much of the results presented here were obtained via a digital simulation model. A program, previously written by the author and noted in the references (1), was used to this end. A listing of this program, and an index of computer runs, may be found in the appendices. A brief description of the simulated satellite is contained in the following section.

SIMULATED SATELLITE

Most of the simulation runs dealt with a satellite with the following characteristics: (1) moments of inertia of 47.5Kg-m^2 (35 slug-ft^2), 67.9Kg-m^2 (50 slug-ft^2), and 33.9Kg-m^2 (25 slug-ft^2) about, respectively, its yaw, roll and pitch axes; (2) a pitch momentum bias of 0.944 Nt-m-sec (0.7 lbf-ft-sec); and (3) switchable control magnets of 10^4 pole-cm strength. There are three such magnets, one each along the yaw, roll and pitch axes. We again note that these are principal axes for the satellite. This satellite is in a circular low altitude orbit with a period of approximately 100 min. We note however, that these parameters of orbit and vehicle are of no special significance, but were chosen merely to implement the digital simulation.

DESCRIPTION OF PROBLEM

In general, a satellite injected into orbit will be observed to be tumbling about with a certain residual angular velocity. This is so in spite of efforts made to release it "gently". Various devices have been utilized to dissipate this residual motion. In our case, the satellite will be launched by a spin stabilized rocket and hence will be inserted into orbit with an angular velocity of roughly 100 RPM about some (known) axis. A simple mechanism (a yo-yo) will then reduce that spin to the order of 1-10 RPM. However, this rate is, in general, still so high as to prevent the basic attitude control system from locking onto some target (the Earth, sun, stars, etc.). A further spin rate reduction is required, preferably to say 0.1 RPM or less. To this end, such active devices as rate gyros, accelerometers, and magnetometers coupled to gas jet systems, and passive devices such as eddy current and magnetic hysteresis rods have been employed.

Reverting to general terms, then, there are two distinct phases of the attitude acquisition procedure. The first is despin, of which we have just spoken, and the second is an attitude orientation phase which facilitates capture by the basic control system.

EQUATIONS OF MOTION

The equations describing the rotary motion of a satellite (or any other rigid body) are most conveniently written for a set of body fixed principal axes. They are the well known Euler's Equations of Motion, with terms added to account for momentum wheels. To wit,

$$\bar{\tau} = \frac{d^{SPACE}}{dt}(\bar{H}) = \frac{d^{BODY}}{dt}(\bar{H}) + \bar{\omega} \times \bar{H} \tag{1}$$

$$\text{where } \bar{H} = \bar{\bar{I}} \cdot \bar{\omega} + \bar{h} \tag{1'}$$

or, in scalar form

$$\tau_1 = \dot{h}_1 + I_1\dot{\omega}_1 + \omega_2 h_3 - \omega_3 h_2 + \omega_2\omega_3(I_3 - I_2) \tag{1a}$$

$$\tau_2 = \dot{h}_2 + I_2\dot{\omega}_2 + \omega_3 h_1 - \omega_1 h_3 + \omega_1\omega_3(I_1 - I_3) \tag{1b}$$

$$\tau_3 = \dot{h}_3 + I_3\dot{\omega}_3 + \omega_1 h_2 - \omega_2 h_1 + \omega_1\omega_2(I_2 - I_1) \tag{1c}$$

The reader is cautioned that great care should be exercised if these axes are not right handed. As mentioned above, these axes are principal axes passing thru the center of mass. Here, the I_i are the moments of inertia about the ith coordinate axis, $\bar{\tau}$ is the applied torque, and $\bar{h}$ is the momentum contribution of the wheels.

CONTROL THEORY

As noted above, the attitude acquisition system performs two principal functions. The first is despinning the tumbling satellite, and the second consists of providing some preferred orientation. Consider the former.

The rotational kinetic energy of the satellite, not including that of any reaction wheels, is

$$T = \frac{1}{2} \sum_i I_i \omega_i^2 \qquad (3)$$

We would like to diminish this quantity.

$$\frac{d}{dt}(T) = \dot{T} = \sum_i I_i \omega_i \dot{\omega}_i = \bar{\tau} \cdot \bar{\omega} \qquad (4)$$

where $\bar{\tau}$ is the external torque acting on the satellite, and $\bar{\tau} \cdot \bar{\omega}$ is the rate at which work is done on the satellite. For a dipole $\bar{M}$ in a magnetic field $\bar{B}$, the torque exerted by the field on the dipole is

$$\bar{\tau} = \bar{M} \times \bar{B} \qquad (5)$$

Combining (4) & (5)

$$\dot{T} = \bar{M} \times \bar{B} \cdot \bar{\omega} = \bar{B} \times \bar{\omega} \cdot \bar{M} \qquad (6)$$

Now consider the quantity $\dfrac{d^{BODY}}{dt}(\bar{B})$

$$\frac{d^{SPACE}}{dt}(\bar{B}) = \frac{d^{BODY}}{dt}(\bar{B}) + \bar{\omega} \times \bar{B} \qquad (7)$$

Suppose $\dfrac{d^{SPACE}}{dt}(\bar{B}) = \bar{0}$. That is, we say that this term is negligible in comparison with the other two terms. Then (7) becomes

$$\frac{d^{BODY}}{dt}(\bar{B}) = \bar{B} \times \bar{\omega} \qquad (8)$$

Substituting in (6) we obtain

$$\dot{T} = \dot{\bar{B}} \cdot \bar{M} \qquad (9)$$

The desired control scheme is now clear. We must simply measure $\dot{\bar{B}}$ along any axis in the satellite, and then change the polarity of a magnet lying along that axis, keeping the sense of $\bar{M}$ opposite that of $\dot{\bar{B}}$. This insures $\dot{T} \le 0$ and decreasing T. The limitations of such a

scheme are also clear. Equation (9) is valid only when (8) is valid, that is, if $\dfrac{d^{SPACE}(\bar{B})}{dt}$ is truly negligible in comparison with $\bar{\omega} \times \bar{B}$. Now, $\bar{B}$ varies in inertia space as $\sin 2\omega_O t$, where ω_O is the orbital frequency. Thus, $\dfrac{d^{SPACE}(\bar{B})}{dt}$ is equivalent to a body rate of $2\omega_O$, and as long as $\bar{\omega}$ is, say, an order of magnitude greater than $2\omega_O$ (about 0.002 rad/sec for low altitude orbits), we may safely neglect $\dfrac{d^{SPACE}(\bar{B})}{dt}$.

For reasons which will become clear later, the chosen control configuration involves three magnets. These lie along the principal axes of the vehicle and are controlled by magnetometers (which measure the field component) along these same axes. The controller operates in a flip-flop manner, switching the magnets' polarity to keep the M_i opposite in sense to $\dot{B}_i$, as measured by magnetometers on board the satellite.

TERMINAL ORIENTATION

We now consider the second phase of attitude acquisition, orientation. We wish to attain some particular attitude. As long as this preferred orientation is with respect to the Earth, any desired attitude may be attained. In this section we will confine our discussion to a satellite having only a pitch momentum bias. Other momentum wheels, if any, are not yet activated at this point. The arguments advanced below are not as rigorous as might be desired but it is felt that their physical appeal more than compensates for that.

Consider the satellite to be orbiting the Earth at angular rate ω_0, and assume also that it has been completely despun. Then it must be that the satellite interprets this situation as though it were spinning backwards at angular velocity $-2\omega_0 \bar{e}_n$, where $\bar{e}_n$ is a unit vector normal to the plane of orbit (and whose sense is determined by the direction of orbit). This is because the satellite senses the Earth's magnetic field passing by it twice per orbit. Note that the trajectory yaw and roll axis components of the Earth's field vary as $\sin 2\theta$, where θ is the angle into orbit from some reference point. Since the system "senses" an angular velocity whose direction is $-\bar{e}_n$, it responds with a corrective torque along the direction $+\bar{e}_n$. The net result of such a torque must be to gradually align the system momentum bias (which properly lies along the vehicle pitch axis) with the orbit normal. In this way the proper yaw-roll attitude is attained. It is the fact that the satellite is not spinning at $2\omega_0$ that results in the net torque along $\bar{e}_n$ which in turn aligns the momentum bias with the orbit normal. But what if it had been spinning at $2\omega_0$ originally, or in some way reached this state. It is apparent that this could occur in only two ways. That is, the system momentum must be either parallel or antiparallel to $\bar{e}_n$. If this is not so, then a component of this momentum lies in the orbit plane and a body fixed torque, also lying in the orbit plane and of the correct magnitude would be required to force the body to rotate about $\bar{e}_n$ at $2\omega_0$. Where is this torque to come from? The author does not believe in its existence and hence concludes that the system momentum bias must lie parallel or antiparallel to $\bar{e}_n$ if it is to rotate at $2\omega_0$. Both of these situations correspond to equilibrium points. We simply note here that the antiparallel situation is not a stable condition.

Our conclusions are then that the satellite will eventually align its momentum bias axis (in our case, the pitch axis) with the orbit normal and thus provide yaw-roll attitude acquisition. If the reader is not completely convinced of this by the rather un-rigorous arguments advanced here, we reassure him by noting that considerable simulation experience is completely in accord with the above analysis. A further discussion concerning the amount of momentum bias required in the above scheme and the effects of more than one momentum wheel, etc., are reserved for later sections. Refer to the section titled "System Performance" and see Appendix C.

STABILITY CONSIDERATIONS

The desired action of the control law just developed is, of course, to reduce the angular velocity of the satellite to zero. Such a condition is not, properly, an equilibrium point for our system, since no provision has been made, as of yet, for a null state for the magnets. They are always on, having one polarity or the other, and the satellite always has some angular acceleration, hence the system possesses no equilibrium points. It would be easy however, to insert deadzones in the magnet controllers. Provided these deadzones are large enough to prevent magnet activation due to the relatively small term $\frac{d\text{SPACE}(\bar{B})}{dt}$ in (7) (which the satellite experiences due to its orbital motion and the rotation of the Earth), then, the condition $\bar{\omega} = 0$, $\bar{B} =$ any, is an equilibrium point. If this sort of provision is not made, then it would seem that the satellite should behave in some sort of quasi-oscillatory manner, at small angular rates. That this condition would, once attained, persist, seems quite plausible from the analysis above. It is not, however, a simple matter to talk about equilibrium points and stability. The equations describing this system are quite intractable to ordinary stability analysis. For one thing, the control law is binary. Even if we replace it with a linear law (the first term of its describing function) the resulting system equations consist of six first order nonlinear differential equations; three involve quadratic combinations, the other three cubic combinations of the state variables (ω_1, ω_2, ω_3, B_1, B_2, B_3).

We should note here that if the system has either a binary control law with deadzones or a linear control law (that is, $\bar{M} = -K\bar{B}$), then there are an infinity of proper equilibrium points. These points belong to one of four distinct classes. However, in order to spare the reader, a discussion of these points and their stability is relegated to Appendix D. Suffice it to say that some of these points (other than $\omega = 0$) can be shown, by analysis, to be neutrally stable. Further, simulation results show that in some domain some of the points are stable.

There are other problems too. We are particularly interested in investigating the existence of any so called "psuedo equilibrium points" (PEP's for brevity). Since $\frac{d\text{SPACE}(\bar{B})}{dt} \neq 0$, and there are various disturbance torques acting on our satellite, and the system equations are nonlinear, there may be points which are not properly equilibrium (stationary) points but near which the system might "hang up". Here we are talking principally about limit cycle behavior, but must also be concerned with various (as yet unknown) forcing functions.

In view of these difficulties, it was decided that digital simulation was the most suitable tool for evaluating this system. A discussion of the results of this simulation work is reserved for the next section and Appendix E.

Although the actual stability analysis is put off until appendices C and E, the conclusions and implications of those analyses are reviewed here. First, we note that, for a vehicle configured as described in the section "Simulated Satellite" (that is to say, the proposed vehicle), the terminal condition (or behavior) corresponds to a stable equilibrium point. This is, of course, most desirable. We again note that the "upside down" orientation (that is, with the pitch axis flipped over) corresponds to an unstable equilibrium state; this too is soothing. Finally we note that, for our vehicle, a spin rate of up to 27.8×10^{-3} radians/sec. about the pitch axis also may be a stable equilibrium state.

This stability conclusion neglects the fact that the direction of $\bar{B}$ in space changes as the satellite orbits the Earth. This turns out to be a saving grace. Nonetheless, for strong enough magnets, the satellite may "track" the field. During such a period it is possible that the despin rate may be reduced by as much as a factor of fifty. For this reason it might be advisable to insert a mode switch for disabling the despin system for an eighth orbit or so if it seems as though this has occured. At the end of that time ω and $\bar{B}$ would no longer be parallel and hence the despin system will again perform properly.

SYSTEM PERFORMANCE

Having convinced the reader (hopefully) that our system works, we now attempt to answer the question - how well? As before, we shall first be concerned with the despin phase.

As the system kinetic energy $\frac{1}{2}\bar{\omega}\cdot\bar{\bar{I}}\cdot\bar{\omega}$ tends towards zero (this was assured in the section titled "Control Theory"), so must $\bar{\omega}$ and the system momentum $\bar{\bar{I}}\cdot\bar{\omega}$ (this, of course, excludes the wheels' momenta). In fact, the despin process may be looked on as a reduction of the momentum term $\bar{\bar{I}}\cdot\bar{\omega}$ to zero. In the very best case this requires a time Δt_{opt} given by

$$\Delta t_{opt} = \frac{|\bar{\bar{I}}\cdot\bar{\omega}|}{\tau_{max}} = \frac{|\bar{P}_O|}{MB} \tag{10}$$

Here $\bar{P}_O = \bar{\bar{I}}\cdot\bar{\omega}_{init}$ is the initial momentum of the body other than that due to the wheels. It is useful to consider an "efficiency factor" α,

$$\alpha = \frac{\Delta t_{opt}}{\Delta t_{actual}} \tag{11}$$

where α of course lies between zero and one. α is a function of various parameters, such as $\bar{\omega}_{init}$, the inertia configuration of the satellite, and the altitude and inclination of orbit. That α is less than one is a reflection of the fact that $\bar{M}$ is not always perpendicular to $\bar{B}$ and that $\bar{\tau}$ does not always lie opposite $\bar{\bar{I}}\cdot\bar{\omega}$.

For a given system configuration ($\bar{\bar{I}}$ & $\bar{M}$) and a given orbit and initial conditions (determining $\bar{\omega}$, $\bar{B}$) we should be able to determine α and hence the time for despin. This was the goal of much of our early simulation, in which various orbits, initial conditions and system configurations were evaluated. This simulation shows that α generally lies in the range 0.55-0.80. For the worst case, a near equatorial orbit with initial spin along the field vector, α may be as poor as 0.15-0.20.

Another way to look at the performance of this system is from an energy point of view. Consider, for simplicity, rotation about a single axis.

$$T = \frac{1}{2}I\omega^2 \tag{12}$$

$$\dot{T} = I\omega\dot{\omega} = \omega\tau \tag{13}$$

$$\therefore I\dot{\omega} = \tau = \alpha MB \qquad (0<\alpha<1) \tag{14}$$

The last of these equations says that ω, and the angular momentum $I\omega$, are always decreasing. The constancy of this rate of decrease depends on the constancy of α. We are of course assuming that τ has the proper sense.

The attitude acquisition part of the problem is more difficult to analyze. If, for example h approaches 0 we will not acquire. But also if h approaches infinity we will acquire, but it will take infinite time. Also, if M approaches 0 or infinity we will not attain the proper attitude. In view of these considerations it would seem that there might be some ratio of momentum bias to magnet torque which would provide optimum performance. By this we mean both minimum time to acquire and minimum roll-yaw error after acquisition. Consider the parameter

$$Tsys = \frac{h}{MB} \tag{15}$$

which has the dimensions of time. It is rather difficult to say anything about this problem analytically, but the following is observed from simulation runs.

We first note that there is a certain minimum bias required for terminal attitude acquisition. It is shown in Appendix C, eq. (C10), that

$$h_O > \Omega \max(I_1, I_2) \tag{C10}$$

for us, $h_O = h + I_3\Omega$ and $\Omega = 2\omega_O$ (for the terminal condition). Thus (C10) becomes

$$h > 2\omega_O\{\max(I_1, I_2) - I_3\} \tag{16}$$

For good performance this amount should be exceeded by a factor of four or more. Little is gained by increasing h beyond this point. If we now choose a particular orbit, the time and quality of acquisition (after despin has been completed) are functions of the magnet strength M. Acquisition time is also, of course, very much a function of attitude at the conclusion of despin. A (very) qualitative picture of this situation is presented in Figure 1. As pointed out previously, for very weak or very strong magnets we do not attain the desired attitude. The author cannot caution the reader too strongly against taking this curve too literally. Much more work would be needed to get a more exact picture of this phenomena. There are too many variables and too few points on the curve of Figure 1 to let the author rest comfortably. As for the quality of roll-yaw acquisition, it is interesting to note that a curve qualitatively similar to Figure 1 may be used to describe this as well. Figure 2 depicts this situation. Note that the minimum point is shifted to about 2000 secs. (This appears to be significant) and the curve is somewhat flatter. The best that we can do appears to be about 2°. Again, the author cannot over-emphasize the tentative nature of these findings. Considerable work remains to be done in this area.

For a well proportioned system, acquisition times are of the order of one orbit. This being the case, it is expected that optimum performance will generally be defined in terms of quality of roll-yaw acquisition rather than in terms of minimum time to acquire.

FIGURE 1

ACQUISITION TIME VS. Tsys

FIGURE 2

ATTITUDE ERROR VS. Tsys
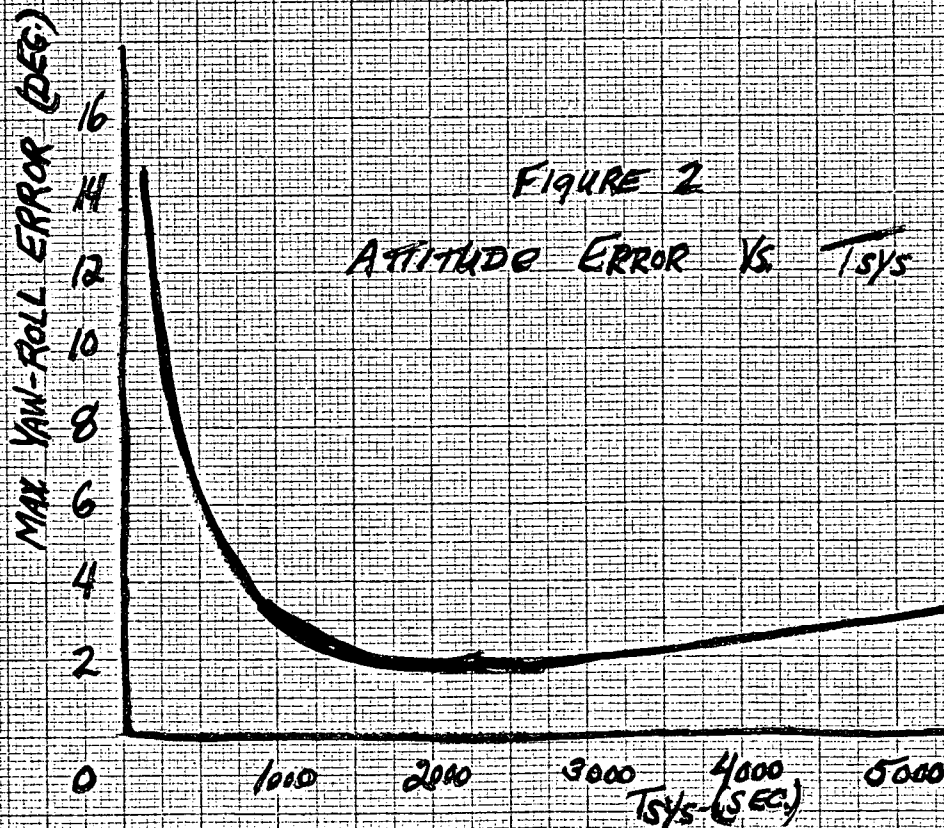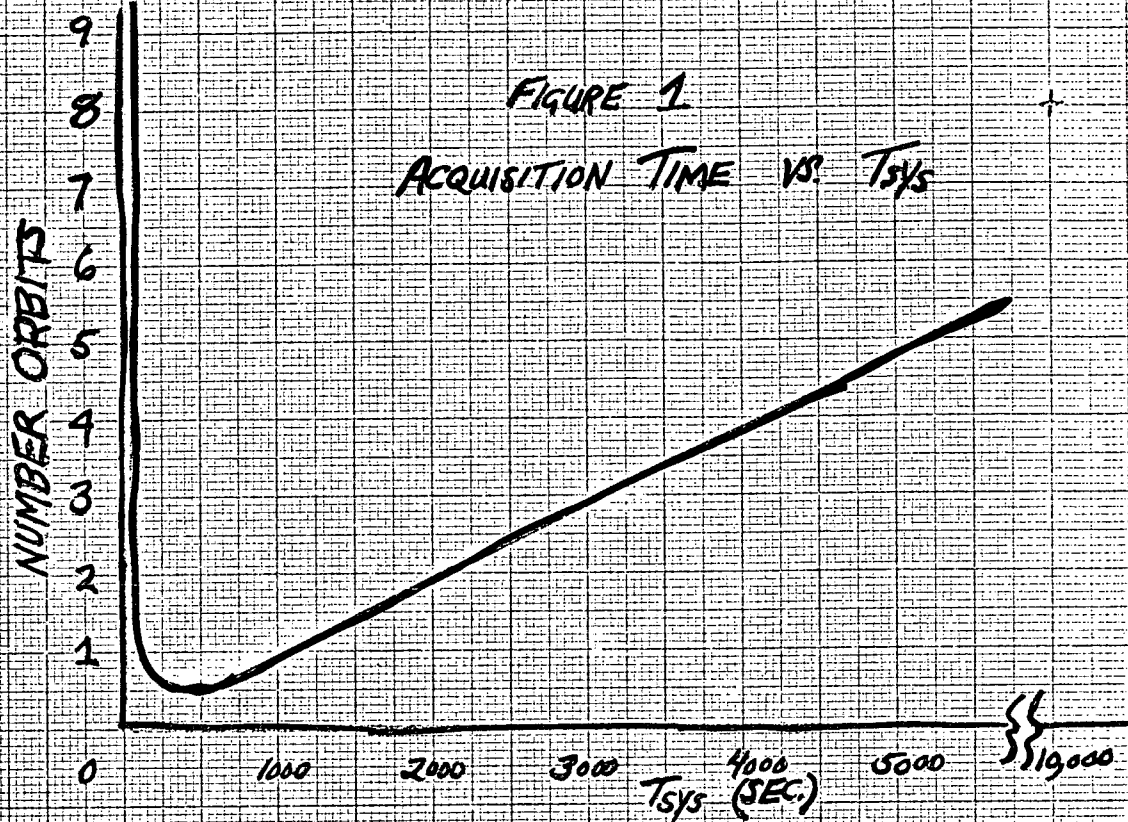
RECOMMENDATIONS FOR FURTHER WORK

The author feels that the work done to date definitely demonstrates the feasibility and desirability of this acquisition control system. Nevertheless, work remains to be done.

There is the matter of system stability. While it seems apparent that there are no stability problems which might interfere with the proper operation of the control system, nonetheless it would be good if we obtained some further analytical results. In particular, an analysis including the effects of the change in $\bar{B}$ due to orbital motion would be interesting.

As noted, we have not included the effects of disturbance torques on the performance of this system. Their effects on both limit cycle behavior and terminal attitude orientation should be investigated. Their neglect has been justified up to this point by the fact that the magnet torques are at least an order of magnitude greater than the disturbances.

Another area which needs refinement is the magnetic field model used in our simulation. So far, only the simple tilted dipole has been used. The effect of a more accurate field model on terminal attitude orientation needs to be investigated.

More investigation into the effects of deadbands, hysteresis loops, and inhibit controls placed in the magnet control loops would be desirable. There is also the matter of crosstalk between magnets and magnetometers. We must be sure that no unstable loop is set up in which switching magnets trigger magnetometers which in turn cause the magnets to switch again.

One other point which will bear further investigation is the potential of using this acquisition system as a complete attitude pointing system. Used with a pitch reaction wheel scanner and simple pitch control loop we would have complete three axis control. Some studies of pointing accuracies versus various system parameters would be useful in investigating this possibility.

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APPENDIX B

DIGITAL SIMULATION PROGRAM LISTING

   This appendix consists of a listing of the digital computer
program used to simulate control system/satellite behavior. The
program is quite similar to the one discussed in reference 1, and
the reader is referred there for a more detailed discussion, flow-
charts, etc. Its operation may be briefly summarized as follows.

   The main program performs the bookkeeping functions of input/
output and maintains overall control. What we are doing here is,
essentially, integrating equation (1), specialized for the vehicle
described under "Simulated Satellite". This is done through sub-
routines WDOT and RK4. An additional problem we must deal with
here is that the applied torques are functions of the vehicle's
attitude. Namely, they are functions of the Earth's magnetic field
components as measured in the vehicle. For this reason we must
keep track of the vehicle's attitude. This is implemented through
the use of Euler Homogeneous parameters in subroutines.EHPQT, NORM,
MAT1, and MULVEC. The Earth's magnetic field is first obtained
in trajectory axis components (subroutine BFLDS1), and then trans-
ferred into body axis components, making use of the direction
cosine matrix (subroutine EHPA). Subroutine TORQUE then computes
the torques arising from the interaction between this field and
the on board magnets. Please note that the subroutine displayed
contains neither deadzones nor hysteresis loops, although some
subroutines incorporating them have been used at times.

```
*JOB
C    ****GENERAL PURPOSE KINEMATIC INTEGRATOR PACKAGE USES EULER        DAV20001
C    ****HOMOGENEOUS PARAMETERS IN ORDER TO SOLVE PROBLEMS INVOLVING    DAV20002
C    ****BODY RATE SPECIFIED ROTATION RATES. *********************      DAV20003
C    ****AUTHOR: ALBERT C. STICKLER                                     DAV20004
C    **********************************************************************DAV20005
C    ****MAIN PROGRAM SKELETON   FILL IN DETAILS, PARAMETERS, ETC.  ******DAV20006
C    ****ASSEMBLAGE INCLUDES SUBROUTINES NORM, MAT1, RK4, EHPA,         DAV20007
C    ****EHPQT, EANGLE, AND INTERP, AMONG OTHERS  ********************DAV20008
C                                                                       DAV20009
      REAL I1, I2, I3, QT(4,4), A(3,3),AXIS(3)                          DAV20010
      REAL*4 INCLIN,BT(3),BB(3)                                         DAV20011
      COMMON/RK/P(4),W(3),QT,DT,WO                                      DAV20012
      COMMON/WDOTS/ T(3), COMM(10), C1, C2, C3, C4, C5, C6              DAV20013
      REAL TIME/0.0/,THETA/0.0/,WE/0.41784E-02/                         DAV20014
      INTEGER*4 I/-1/, IHEAD/-1/                                        DAV20015
      READ(5,90)I1,I2,I3,BIAS,FACTOR                                    DAV20016
      READ(5,90)   PRINT,BSTEP                                          DAV20017
      READ(5,90)W                                                       DAV20018
      READ(5,90)DT, TRUN                                                DAV20019
      READ(5,90)INCLIN,EARTHR,ALT                                       DAV20020
      PERIOD=0.3151263*SQRT(0.1*ALT)**3                                 DAV20021
      WRITE(6,110) I1,I2,I3,BIAS,FACTOR                                 DAV20022
      WRITE(6,111) PRINT,BSTEP                                          DAV20023
      WRITE(6,112) DT,INCLIN,EARTHR,ALT,PERIOD                          DAV20024
      C1=(I2-I3)/I1                                                     DAV20025
      C2=(I3-I1)/I2                                                     DAV20026
      C3=(I1-I2)/I3                                                     DAV20027
      C4=1/I1                                                           DAV20028
      C5=1/I2                                                           DAV20029
      C6=1/I3                                                           DAV20030
C  READ AND WRITE INITIAL ATTITUDE                                      DAV20031
      READ(5,90)P                                                       DAV20032
      WRITE(6,91)P                                                      DAV20033
   91 FORMAT(//,'  INITIAL EHPS HERE', 4F10.4)                          DAV20034
      CONST=0.001*FACTOR                                                DAV20035
      COMM(1)=BIAS*C4                                                   DAV20036
      COMM(2)=BIAS*C5                                                   DAV20037
      IBSTEP=BSTEP/360.0*PERIOD/DT                                      DAV20038
      IPRINT=PRINT                                                      DAV20039
      DTHETA=360.0/PERIOD*DT                                            DAV20040
      CALL BFLDS1(INCLIN,ALT,BT)                                        DAV20041
      CALL BFLDS2(EARTHR,THETA,BT)                                      DAV20042
      WO=6.2832/PERIOD                                                  DAV20043
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34          DTHAF=DT*0.5
35       50 CALL NORM(P,PMAG)                                          DAV20044
36          I=I+1                                                      DAV20045
37          IF(I/IPRINT*IPRINT .NE. I) GO TO 75                        DAV20046
38          ERROR=PMAG-1.0                                             DAV20047
39          ENERGY=I1*W(1)*W(1)+I2*W(2)*W(2)+I3*W(3)*W(3)              DAV20048
40          IHEAD=IHEAD+1                                              DAV20049
41          IF(IHEAD/50*50 .EQ. IHEAD) WRITE(6,101)                    DAV20050
42          WRITE(6,100) TIME,W,BB,ERROR,ENERGY,A(1,3),A(2,3)          DAV20051
43       75 TIME=TIME+DT                                               DAV20052
44          CALL EHPA(P,A)                                             DAV20053
45          THETA=THETA+DTHETA                                         DAV20054
46          IF(I/IBSTEP*IBSTEP .EQ. I)CALL BFLDS2(EARTHR+WE*TIME,THETA,BT) DAV20055
47          IF(ENERGY .LT.  0.01) CALL BFLDS3(THETA,BT)                DAV20056
48          CALL MULVEC(A,BT,BB)                                       DAV20057
49          CALL TORQUE(T,BB,CONST)                                    DAV20058
                                                                       DAV20059
50          CALL RK4                                                   DAV20060
51          IF(TIME .GT. TRUN) STOP                                    DAV20061
52          GO TO 50                                                   DAV20062
53       90 FORMAT(5F10.3)                                             DAV20063
54      100 FORMAT(1H ,F6.0, 2X,3(E10.3,1X),3(F6.3,1X), 2E15.5,2F7.3)  DAV20064
55      101 FORMAT(1H1,' TIME       OMEGA1       OMEGA2       OMEGA3       B1      BDAV20065
           62      B3          ERROR        ENERGY     +ROLL  NEG. YAW') DAV20066
56     1100FORMAT(//////,50X,'SYSTEM PARAMETERS ARE',//,60X,'INERTIA(1)=', DAV20067
           1   F10.2,//,60X,'INERTIA(2)=',F10.2,//,60X,'INERTIA(3)=',F10.2, DAV20068
           2   //,60X,'MOMENTUM BIAS=',F10.3,//,60X,'MAGNET STRENGTH FACTOR=',DAV20069
           3   F10.2)                                                   DAV20070
57     1110FORMAT(//////,50X,'INTERVALS ARE',//,60X,'PRINT EVERY',F10.0, DAV20071
           1   '   TIME STEPS',//,25X,'CALCULATE MAGNETIC FIELD EVERY',F10.2, DAV20072
           2   '    DEGREES OF ORBIT')                                  DAV20073
58     1120FORMAT(//////,10X,'INTEGRATION STEP SIZE=',F5.2,'   SECONDS.-- RUNGDAV20074
           1E-KUTTA FOURTH ORDER INTEGRATION WITH EULER HOMOGENEOUS PARAMETERSDAV20075
           2',////,50X,'ORBIT SPECIFICATIONS',//,25X,'INCLINATION ANG.=',F10.2DAV20076
           3   ,' DEGREES',//,25X,'ROTATION OF EARTH MINUS LONGITUDE OF THE DAV20077
           4   NODES(INITIAL)=',F10.2,'   DEGREES',//,50X,'ORBIT ALTITUDE (CENDAV20078
           5TRAL DISTANCE)=',F10.0,'   KM.',//,25X,'PERIOD FOR THIS ORBIT=', DAV20079
           6   F10.0,'   SECS.',///,25X,'ALL UNITS ARE IN M-K-S SYSTEM UNLESS DAV20080
           7   OTHERWISE NOTED')                                        DAV20081
59          END                                                        DAV20082
```

```
60          SUBROUTINE EHPA(P,A)                                    DAV20083
61          REAL*4 P(4), A(3,3)                                    DAV20084
      C  ****THIS ROUTINE RETURNS THE ROTATION MATRIX A CORRESPONDING   DAV20085
      C  ****TO THE EULER HOMOGENEOUS PARAMETERS P SUB I,  A*V(OLD)=V(NEW) DAV20086
62          P12=P(1)*P(1)                                          DAV20087
63          P22=P(2)*P(2)                                          DAV20088
64          P32=P(3)*P(3)                                          DAV20089
65          P42=P(4)*P(4)                                          DAV20090
66          A(1,1)=P42+P12-P22-P32                                 DAV20091
67          A(1,2)=2.*(P(1)*P(2)+P(3)*P(4))                        DAV20092
68          A(1,3)=2.*(P(1)*P(3)-P(2)*P(4))                        DAV20093
69          A(2,1)=2.*(P(1)*P(2)-P(3)*P(4))                        DAV20094
70          A(2,2)=P42+P22-P32-P12                                 DAV20095
71          A(2,3)=2.*(P(2)*P(3)+P(1)*P(4))                        DAV20096
72          A(3,1)=2.*(P(1)*P(3)+P(2)*P(4))                        DAV20097
73          A(3,2)=2.*(P(2)*P(3)-P(1)*P(4))                        DAV20098
74          A(3,3)=P42+P32-P22-P12                                 DAV20099
75          RETURN                                                 DAV20100
76          END                                                    DAV20101


77          SUBROUTINE EHPQT(P,QT)                                 DAV20102
78          REAL*4 P(4), QT(4,4)                                   DAV20103
      C  ****THIS SUBROUTINE SETS UP THE MATRIX Q TRANSPOSE FOR ANY GIVEN SET DAV20104
      C  ****OF EULER'S HOMOGENEOUS PARAMETERS--- THE P SUB I          DAV20105
79          QT(1,1)=P(4)                                           DAV20106
80          QT(1,2)=-P(3)                                          DAV20107
81          QT(1,3)=P(2)                                           DAV20108
82          QT(2,1)=P(3)                                           DAV20109
83          QT(2,2)=P(4)                                           DAV20110
84          QT(2,3)=-P(1)                                          DAV20111
85          QT(3,1)=-P(2)                                          DAV20112
86          QT(3,2)=P(1)                                           DAV20113
87          QT(3,3)=P(4)                                           DAV20114
88          QT(4,1)=-P(1)                                          DAV20115
89          QT(4,2)=-P(2)                                          DAV20116
90          QT(4,3)=-P(3)                                          DAV20117
91          RETURN                                                 DAV20118
92          END                                                    DAV20119
```

```
109          SUBROUTINE NORM(P,MAG)                                      DAV20140
110          REAL*4 P(4),MAG                                             DAV20141
     C  ****THIS ROUTINE NORMALIZES P TO LENGTH = UNITY AND RETURNS ITS   DAV20142
     C  ****LENGTH PRIOR TO NORMALIZATION AS A CONVENIENT ERROR MEASURE.  DAV20143
111          MAG=0.                                                      DAV20144
112          DO 1 I=1,4                                                  DAV20145
113        1 MAG=MAG+P(I)*P(I)                                           DAV20146
114          MAG=SQRT(MAG)                                               DAV20147
115          DO 2 I=1,4                                                  DAV20148
116        2 P(I)=P(I)/MAG                                               DAV20149
117          RETURN                                                      DAV20150
118          END                                                         DAV20151
```

```
93           SUBROUTINE WDOT(W,WD)                                       DAV20120
     C  EQUATIONS OF MOTION OF OUR BODY                                   DAV20121
     C  WD'S ARE OMEGA DERIVATIVES; T'S ARE TORQUES, C'S AND COMM'S ARE CONSTDAV20122
     C  AND MOMENTUM BIASES                                               DAV20123
94           REAL*4 W(3), WD(3)                                          DAV20124
95           COMMON/WDOTS/ T(3), COMM(10), C1, C2, C3, C4, C5, C6        DAV20125
96           WD(1)=T(1)*C4+C1*W(2)*W(3) -COMM(1)*W(2)                    DAV20126
97           WD(2)=T(2)*C5+C2*W(1)*W(3) +COMM(2)*W(1)                    DAV20127
98           WD(3)=T(3)*C6+C3*W(1)*W(2)                                  DAV20128
99           RETURN                                                      DAV20129
100          END                                                         DAV20130
```

```
101          SUBROUTINE MAT1(A,V1,V2)                                    DAV20131
     C  UTILITY MULTIPLICATION-- USED IN RELATING OMEGA'S TO P DOTS       DAV20132
102          REAL*4 A(4,4), V1(3), V2(4)                                 DAV20133
103          DO 1 I=1,4                                                  DAV20134
104          V2(I)=0.                                                    DAV20135
105          DO 1 J=1,3                                                  DAV20136
106        1 V2(I)=V2(I)+A(I,J)*V1(J)                                    DAV20137
107          RETURN                                                      DAV20138
108          END                                                         DAV20139
```

```
119          SUBROUTINE RK4                                              DAV20152
120          COMMON/RK/ P(4),W(3),QT(4,4),DT,WO                          DAV20153
121          REAL*4 PDO(4),D(4)                                          DAV20154
122          REAL*4 D1(3),D2(3),D3(3),D4(3),S(3),PD1(4),PD2(4),PD3(4),PD4(4)  DAV20155
      C    ****THIS SUBROUTINE INTEGRATES W (BODY REFERENCED ANGULAR VELOCITY)  DAV20156
      C    ****AND P (THE EULER HOMOGENEOUS PARAMETERS) FROM TIME T TO T+DT.    DAV20157
      C    ****TECHNIQUE IS A FOURTH ORDER RUNGE KUTTA SCHEME.          DAV20158
      C    *************************************************************************DAV20159
      C    ****REQUIRES SUBROUTINES WDOT (USER SUPPLIED), MAT1, AND ARRAY QT.   DAV20160
123          CALL EHPQT(P,QT)                                            DAV20161
124          CALL MAT1(QT,W,PD1)                                         DAV20162
125          CALL WDOT(W,D1)                                             DAV20163
126          DO 10 I=1,3                                                 DAV20164
127     10 S(I)=W(I)+0.5*DT*D1(I)                                       DAV20165
128          DO 11 I=1,4                                                 DAV20166
129     11 D(I)=P(I)+0.5*DT*PD1(I)                                      DAV20167
130          CALL EHPQT(D,QT)                                            DAV20168
131          CALL MAT1(QT,S,PD2)                                         DAV20169
132          CALL WDOT(S,D2)                                             DAV20170
133          DO 20 I=1,3                                                 DAV20171
134     20 S(I)=W(I)+0.5*DT*D2(I)                                       DAV20172
135          DO 21 I=1,4                                                 DAV20173
136     21 D(I)=P(I)+0.5*DT*PD2(I)                                      DAV20174
137          CALL EHPQT(D,QT)                                            DAV20175
138          CALL MAT1(QT,S,PD3)                                         DAV20176
139          CALL WDOT(S,D3)                                             DAV20177
140          DO 30 I=1,3                                                 DAV20178
141     30 S(I)=W(I)+      DT*D3(I)                                     DAV20179
142          DO 31 I=1,4                                                 DAV20180
143     31 D(I)=P(I)+      DT*PD3(I)                                    DAV20181
144          CALL EHPQT(D,QT)                                            DAV20182
145          CALL MAT1(QT,S,PD4)                                         DAV20183
146          CALL WDOT(S,D4)                                             DAV20184
147          DO 40 I=1,3                                                 DAV20185
148     40 W(I)=W(I)+DT*(D1(I)+2.*(D2(I)+D3(I))+D4(I))/6.              DAV20186
149          PDO(1)=+WO*P(2)                                             DAV20187
150          PDO(2)=-WO*P(1)                                             DAV20188
151          PDO(3)=-WO*P(4)                                             DAV20189
152          PDO(4)=+WO*P(3)                                             DAV20190
***ERROR***    MO-0
153          DO 50 I=1,4                                                 DAV20191
154     50 P(I)=P(I)+DT*(PD1(I)+PD4(I)+2.*PD2(I)+2.*PD3(I)+6.*PDO(I))/12.0  DAV20192
155          RETURN                                                      DAV20193
156          END                                                         DAV20194
```

```
190        SUBROUTINE TORQUE(T,B,CONST)                                        DAV20233
    C  IDEAL MAGNET SWITCHER                                                   DAV20234
    C  SETS MAGNET POLARITIES OPPOSITE IN SIGN TO B DOTS                       DAV20235
    C  SIGN OF B DOTS IS DETERMINED BY COMPARING PRESENT (B'S) VS. PREVIOUS    DAV20236
    C  (BOLD'S) BODY COMPONENTS OF MAGNETIC FIELD                              DAV20237
    C  THEN, TORQUE= M CROSS B                                                 DAV20238
191        REAL*4 T(3), B(3), M1, M2, M3, B1OLD/0.0/, B2OLD/0.0/, B3OLD/0.0/   DAV20239
192        B1=B(1)                                                            DAV20240
193        B2=B(2)                                                            DAV20241
194        B3=B(3)                                                            DAV20242
195        BD1NEG=B1OLD-B1                                                     DAV20243
196        BD2NEG=B2OLD-B2                                                     DAV20244
197        BD3NEG=B3OLD-B3                                                     DAV20245
198        M1=SIGN(CONST, BD1NEG)                                             DAV20246
199        M2=SIGN(CONST, BD2NEG)                                             DAV20247
200        M3=SIGN(CONST, BD3NEG)                                             DAV20248
201        T(1)=M2*B3-M3*B2                                                    DAV20249
202        T(2)=M3*B1-M1*B3                                                    DAV20250
203        T(3)=M1*B2-M2*B1                                                    DAV20251
204        B1OLD=B1                                                            DAV20252
205        B2OLD=B2                                                            DAV20253
206        B3OLD=B3                                                            DAV20254
207        RETURN                                                             DAV20255
208        END                                                               DAV20256


157        SUBROUTINE MULVEC(A,V1,V2)                                         DAV20195
158        REAL*4 A(3,3), V1(3), V2(3)                                        DAV20196
    C  MULTIPLIES A*V1=V2                                                      DAV20197
159        DO 1 I=1,3                                                          DAV20198
160        V2(I)=0.0                                                           DAV20199
161        DO 1 K=1,3                                                          DAV20200
162      1 V2(I)=V2(I)+A(I,K)*V1(K)                                            DAV20201
163        RETURN                                                             DAV20202
164        END                                                               DAV20203
```

```
       SUBROUTINE BFLDS1(INCLIN,A,B)                                    DAV20204
165
C   RETURNS MAGNETIC FIELD FOR VARIOUS POSITIONS IN ORBIT-- SEE SEPARATE DAV20205
                                                                        DAV20206
C   DOCUMENTATION
       REAL    B(3), INCLIN, PI/3.1416/                                 DAV20207
166
       RADDEG=PI/180.                                                   DAV20208
167
       BNOM=1.011E+12/(4.0*PI*A*A*A)                                    DAV20209
168
       SN=0.2026                                                        DAV20210
169
       CN=0.980                                                         DAV20211
170
       SI=SIN(INCLIN*RADDEG)                                            DAV20212
171
       CI=COS(INCLIN*RADDEG)                                            DAV20213
172
       RETURN                                                           DAV20214
173
       ENTRY BFLDS2(EARTHR,THETA,B)                                     DAV20215
174
C   THETA IS THE ANGLE IN ORBIT FORWARD FROM ASCENDING NODE (DEGREES)   DAV20216
C   EARTHR= W(EARTH)*T- LONGITUDE OF THE ASCENDING NODE  --(ALSO DEGREES)DAV20217
                                                                        DAV20218
175    U=(69.0-EARTHR)*RADDEG
       CU=COS(U)                                                        DAV20219
176
       SU=SIN(U)                                                        DAV20220
177
       B(3)=BNOM*(CI*CN+SI*SU*SN)                                       DAV20221
178
       C1=-CU*SN                                                        DAV20222
179
       C2=CI*SU*SN-SI*CN                                                DAV20223
180
   10  V=THETA*RADDEG                                                   DAV20224
181
       CV=COS(V)                                                        DAV20225
182
       SV=SIN(V)                                                        DAV20226
183
       B(1)=2.0*BNOM*(C1*CV+C2*SV)                                      DAV20227
184
       B(2)=BNOM*(C1*SV-C2*CV)                                          DAV20228
185
       RETURN                                                           DAV20229
186
       ENTRY BFLDS3(THETA,B)                                            DAV20230
187
       GO TO 10                                                         DAV20231
188
       END                                                              DAV20232
189
```

APPENDIX C

DUAL SPIN EQUILIBRIUM CONSIDERATIONS

In this appendix we consider the equilibrium and stability of a dual spin configuration. Although this work has been done before (e.g. reference 6), we derive the results here in a simpler way. These results were obtained independently by the author before his acquaintance with reference 6 and are included here for completeness.

Consider a body with a single momentum bias wheel; the wheel spins at constant rate and carries angular momentum $\bar{h} = \{0,0,h\}$. The body is spinning about the same axis as is the rotor; for definiteness this is taken as the "3" axis. Suppose we consider the torque free motion of the body in the "neighborhood" of an equilibrium point, namely

$$\omega_1 = \omega_2 = 0, \quad \omega_3 = \Omega \tag{C1}$$

In the neighborhood of this point

$$\omega_1 = \delta\omega_1, \quad \dot{\omega}_1 = \delta\dot{\omega}_1, \tag{C2,a}$$

$$\omega_2 = \delta\omega_2, \quad \dot{\omega}_2 = \delta\dot{\omega}_2 \tag{C2,b}$$

$$\omega_3 = \Omega + \delta\omega_3, \quad \dot{\omega}_3 = \delta\dot{\omega}_3 \tag{C2,c}$$

Now, the equations of motion, adapted from eq. (1) are

$$I_1\dot{\omega}_1 + \omega_2\{h + \omega_3(I_3 - I_2)\} = 0 \tag{C3,a}$$

$$I_2\dot{\omega}_2 - \omega_1\{h + \omega_3(I_3 - I_1)\} = 0 \tag{C3,b}$$

$$I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) = 0 \tag{C3,c}$$

Substituting eq. (C2) into these, and dropping second order terms (involving products of the $\delta\omega$'s), we obtain

$$I_1\delta\dot{\omega}_1 + \delta\omega_2\{h + (\Omega + \delta\omega_3)(I_3 - I_2)\} = 0 \tag{C4,a}$$

$$I_2\delta\dot{\omega}_2 - \delta\omega_1\{h + (\Omega + \delta\omega_3)(I_3 - I_1)\} = 0 \tag{C4,b}$$

$$I_3\delta\dot{\omega}_3 = 0 \tag{C4,c}$$

Thus we see immediately $\delta\omega_3 = 0$, therefore $\omega_3 = \Omega = $ constant. Then, equations a and b can be combined to yield

$$\delta\ddot{\omega}_1 + \frac{\delta\omega_1}{I_1 I_2}\{h + \Omega(I_3 - I_2)\}\{h + \Omega(I_3 - I_2)\} = 0 \tag{C5}$$

What we have obviously obtained here is the equation for a simple harmonic oscillator. Consider one special case first. If Ω is "small", then ω_1 (and ω_2, which has the same form of solution) oscillates with circular frequency

$$\lambda_0 = \frac{h}{\sqrt{I_1 I_2}} \tag{C6}$$

If Ω is not small, let us define an equivalent inertia

$$I_{3e} = I_3 + h/\Omega \tag{C7}$$

Then eq. (C5) reads

$$\delta\ddot{\omega}_1 + \frac{\delta\omega_1 \Omega^2 (I_{3e} - I_2)(I_{3e} - I_1)}{I_1 I_2} = 0 \tag{C8}$$

It is clear that if either

$$I_{3e} > I_1 \text{ and } I_{3e} > I_2 \tag{C9,a}$$

or

$$I_{3e} < I_1 \text{ and } I_{3e} < I_2 \tag{C9,b}$$

that is, if I_{3e} is a minimum or maximum inertia, then, our system is stable in the Liapunov sense. On the other hand, if I_{3e} is an intermediate inertia, the system is unstable. These results are of a physically appealing nature since they are of the same form as the results for spinning <u>rigid</u> bodies. In fact, if in eq. (C7) $h = 0$, $I_{3e} = I_3$, and those results are obtained directly.

For non-rigid (i.e., damped) bodies, these requirements are modified somewhat. First, there is generally little damping on the rotor, and that which is present has a destabilizing effect (almost always). Consider then a system with negligible damping on the rotor and a non-negligible amount on the main body. If the spin directions are such that the momenta of rotor and main body add ($\Omega > 0$), it can be shown that it is necessary and sufficient for stability that I_{3e} be the major principal axis (maximum). Conversely, if the momenta oppose each other ($\Omega < 0$), it is necessary and sufficient that

$$h_0 = h + I_3\Omega > 0 \tag{C10}$$

$$\frac{-h}{I_3} < \Omega < 0 \tag{C11}$$

There is one additional region of stability for this system, and it exists only if I_3 is a major axis. The region defined by

$$\Omega < \frac{-h}{I_3 - \max(I_1, I_2)} \tag{C12}$$

corresponds to I_{3e} being a major principal axis while $\Omega < 0$. As $h \rightarrow 0$ the above results reduce to the simple major spin axis requirements.

Definition (C7) may be combined with this observation into a simple rule, to wit

$$h_O > I\Omega \qquad\qquad\qquad (C10)$$

where h_O is the system momentum bias (equal to $h + I_3\Omega$), and $I = \max(I_1, I_2)$. This same conclusion is obtained in reference 6 (page A-82).

The result (C10) has a direct application to the attitude acquisition phenomena noted under the section "Terminal Orientation". Namely, since the magnets will drive the satellite at $2\omega_O$ in the terminal orientation, and since we wish the terminal attitude to correspond to a stable condition, it is necessary that (C10) hold for $\Omega = 2\omega_O$. For a given orbit (which determines ω_O) and given set of inertias, this requirement then specifies a minimum h. The validity of this conclusion is supported by a number of simulation runs. In fact, it was observed that for good terminal attitude acquisition performance the left side of (C10) should be at least three or four times the right side.

APPENDIX D

SYSTEM EQUILIBRIUM POINTS

Consider the equilibrium states of the acquisition system described in this report under two conditions. (a) The magnetic field is fixed in trajectory axis coordinates, that is

$$\frac{d^{SPACE}}{dt}(\bar{B}) = 0 \tag{D1}$$

and (b), the magnet controllers employ a linear law

$$\bar{M} = -K\dot{\bar{B}} \tag{D2}$$

where $\dot{\bar{B}}$ is the time rate of change of $\bar{B}$ as measured in the vehicle. As mentioned in the body of this report, under the section titled "Stability", this system has no equilibrium points unless some modification is made to the flip-flop magnet control law. Now, let us consider the equilibrium points possible under the two conditions above.

The behavior of our system may be completely characterized by the six state variables ω_1, ω_2, ω_3, B_1, B_2, B_3 . Equilibrium points may be identified by setting the time derivatives of these variables to zero and solving the resulting six equations in six unknowns. That is, we require

$$\dot{\bar{\omega}} = 0 \tag{D3}$$

$$\dot{\bar{B}} = \bar{B} \times \bar{\omega} = 0 \qquad\qquad eq.(8)=(D4)$$

Starting with equation (1), we make the following substitutions: $\bar{\tau} = \underline{0}$ (torque free motion), $\dot{\bar{h}} = 0$ (constant speed wheel), and eq. (1') $\bar{H} = \bar{\bar{I}}\cdot\bar{\omega} + \bar{h}$. We then obtain

$$\bar{\bar{I}}\cdot\dot{\bar{\omega}} = -\bar{\omega} \times (\bar{\bar{I}}\cdot\bar{\omega} + \bar{h}) = 0 \tag{D5}$$

The right hand side of (D5) equals zero according to (D3). An equilibrium point must then, by definition, satisfy equations (D4) and (D5). (D4) simply requires that

$$\bar{\omega} = \alpha(t)\bar{B} \tag{D6}$$

where $\alpha(t)$ may or may not be zero and may or may not be constant. Now, if

$$\bar{h} = \{0, 0, h\} = h \tag{D7}$$

That is, we have a pitch wheel only, then (D5) becomes, in scalar form

$$\omega_2\omega_3(I_3 - I_2) + h\omega_2 = 0 \tag{D8,a}$$

(D5, modified)

$$\omega_1\omega_3(I_1 - I_3) - h\omega_1 = 0 \tag{D8,b}$$

$$\omega_1\omega_2(I_2 - I_1) = 0 \tag{D8,c}$$

From the above discussion we now obtain the following results

(I) $\bar{\omega} = \{0, 0, 0\}$, $\bar{B}$ = any, $\alpha = 0$ is an equilibrium point, since it clearly satisfies eq. (D4) & (D8).

(II) $\bar{\omega} = \{0$, any, $h/(I_2 - I_3)\}$, $\bar{B} \parallel \bar{\omega}$ is an equilibrium point.

(III) $\bar{\omega} = \{$any, 0, $h/(I_1 - I_3)\}$, $\bar{B} \parallel \bar{\omega}$ is an equilibrium point.

(IV) $\bar{\omega} = \{0, 0$, any$\}$, $\bar{B} \parallel \bar{\omega}$ is the last equilibrium point.

These four classes contain all the equilibrium points of this system. In passing I note (for the mathematicians among us) that there are an infinity of equilibrium points. Note also that at equilibrium points, $\dot{\bar{B}} = 0$ so $\bar{M} = 0$ and so the fact that we set $\bar{\tau} = 0$ at the outset does not affect the validity of the results obtained here.

There is, curiously, an alternate method of obtaining these same results. Starting with the equation of motion (1), the control law (D2), and the assumptions (D1) and (D7), we have

$$\tau_1 = I_1\dot{\omega}_1 + \omega_2 h \qquad\qquad\qquad\qquad (D9,a)$$

$$\tau_2 = I_2\dot{\omega}_2 - \omega_1 h$$

$$\tau_3 = I_3\dot{\omega}_3$$

where we have dropped the second order terms involving products of the ω's. Using eq. (D2) and (D4), we obtain

$$M_1 = K(\omega_2 B_3 - \omega_3 B_2) \qquad\qquad\qquad (D10,a)$$

$$M_2 = K(\omega_3 B_1 - \omega_1 B_3) \qquad\qquad\qquad (D10,b)$$

$$M_3 = K(\omega_1 B_2 - \omega_2 B_1)$$

Remembering eq. (5)

$$\bar{\tau} = \bar{M} \times \bar{B} \qquad\qquad\qquad\qquad\qquad (5)$$

and substituting $\bar{\tau}$ into eq. (D9), we obtain

$$I_1\dot{\omega}_1 + \omega_2 h = B_3 K(\omega_3 B_1 - \omega_1 B_3) - B_2 K(\omega_1 B_2 - \omega_2 B_1) \qquad (D11,a)$$

$$I_2\dot{\omega}_2 - \omega_1 h = B_1 K(\omega_1 B_2 - \omega_2 B_1) - B_3 K(\omega_2 B_3 - \omega_3 B_2) \qquad (D11,b)$$

$$I_3\dot{\omega}_3 = B_2 K(\omega_2 B_3 - \omega_3 B_2) - B_1 K(\omega_3 B_1 - \omega_1 B_3) \qquad (D11,c)$$

Using the definition of equilibrium points

$$\dot{\bar{\omega}} = 0$$

(here we are considering only $\bar{\omega}$ as a state variable, not $\bar{B}$)

we obtain three <u>linear</u> <u>homogeneous</u> equations in the three unknowns - ω_1, ω_2, and ω_3. For a solution to exist, the determinant of the coefficients must equal zero. That is

$$\begin{vmatrix} -(B_2^2 + B_3^2) & (B_1B_2 - h/K) & B_1B_3 \\ (B_1B_2 + h/K) & -(B_1^2 + B_3^2) & B_2B_3 \\ B_1B_3 & B_2B_3 & -(B_1^2 + B_2^2) \end{vmatrix} = 0 \qquad (D12)$$

This equation reduces to

$$\frac{-h^2}{K^2}(B_1^2 + B_2^2) = 0 \qquad (D12')$$

or

$$\bar{B} = \{0, 0, B_3\} \qquad (D13)$$

Substituting this back into eq. (D11) we have

$$I_1\dot{\omega}_1 + \omega_2 h = -B_3^2 K \omega_1 \qquad (D14,a)$$

$$I_2\dot{\omega}_2 - \omega_1 h = -B_3^2 K \omega_2 \qquad (D14,b)$$

$$I_3\dot{\omega}_3 = 0 \qquad (D14,c)$$

Since $\dot{\bar{\omega}} = 0$, we obtain the following equilibrium condition

$$\omega = \{0, 0, \text{any}\} \qquad (D15)$$

This result is not valid if ω_3 is large, since we neglected $\omega_3(I_3 - I_2)$ and $\omega_3(I_3 - I_1)$ in comparison to h in writing eq. (D9). To the extent that this is valid, (D15) is correct. Note that (D15) is the same as condition (IV) identified previously, and that (I) is a special case of (IV).

The only special thing about this alternate derivation is that we did not assume $\dot{\bar{B}} = 0$ at the outset. We were considering cases where $\bar{\omega} = 0$ but $\bar{B}$ and hence $\bar{\tau}$ not necessarily equal to zero. Note that, in retrospect, we now have $\bar{B} = \{0, 0, B_3\}$, so $\dot{B}_1 = \dot{B}_2 = M_1 = M_2 = 0$. The result is then that $\bar{\tau} = 0$ anyway.

In the next section we consider the stability of the equilibrium points enumerated here.

APPENDIX E

STABILITY OF EQUILIBRIUM POINTS

In this appendix we investigate the stability of a particular equilibrium point, namely point IV of Appendix D. This stability investigation is carried out by linearizing the system equations about the equilibrium point and then forming the characteristic equation. A study of the roots of this equation then determines the stability of the system near the equilibrium point. This does not rule out the possibility of limit cycle like behavior but is nonetheless a useful technique. From the analysis that follows, certain conclusions may be drawn. These conclusions and their consequences are discussed at the end of this appendix.

Consider perturbations about the equilibrium point

$$\bar{\omega} = \{0,\ 0,\ \Omega\} \tag{E1}$$

$$\bar{B} = \{0,\ 0,\ B\}$$

Then

$$\bar{\omega} = \{\delta\omega_1,\ \delta\omega_2,\ \Omega + \delta\omega_3\} \tag{E2}$$

$$\bar{B} = \{\delta B_1,\ \delta B_2,\ B + \delta B_3\}$$

The following equations then apply

$$\dot{\bar{B}} = \bar{B} \times \bar{\omega} \tag{8}$$

$$\delta\dot{B}_1 = \Omega\delta B_2 - B\delta\omega_2$$

$$\delta\dot{B}_2 = B\delta\omega_1 - \Omega\delta B_1 \tag{E3}$$

$$\delta\dot{B}_3 = 0$$

where we have dropped second order terms in eq. (E3). Now, assuming a linear control law, i.e.,

$$\bar{M} = -K\dot{\bar{B}} \tag{E4}$$

$$M_1 = K(B\delta\omega_2 - \Omega\delta B_2) \tag{E5}$$

$$M_2 = K(\Omega\delta B_1 - B\delta\dot{\omega}_1)$$

$$M_3 = 0$$

Then, since

$$\bar{\tau} = \bar{M} \times \bar{B} \tag{5}$$

We obtain

$$\tau_1 = KB(\Omega\delta B_1 - B\delta\omega_1) \tag{E6}$$

$$\tau_2 = KB(\Omega\delta B_2 - B\delta\omega_2)$$

$$\tau_3 = 0$$

E-1

Similarly, we may start with eq. (1), substitute from (E2) and drop second order terms in the result. Thus we obtain the linearized equations of motion about the equilibrium point

$$I_1 \delta\dot\omega_1 = \tau_1 - \delta\omega_2(h + \Omega(I_3 - I_2)) \tag{E7}$$

$$I_2 \delta\dot\omega_2 = \tau_2 + \delta\omega_1(h + \Omega(I_3 - I_1))$$

$$I_3 \delta\dot\omega_3 = \tau_3$$

These may be written in a way that makes them easier to interpret by using the following shorthand notation:

$$I_{3e} = \frac{h}{\Omega} + I_3 \tag{E8}$$

$$I_{3e}\Omega = h + I_3\Omega = ho \tag{E9}$$

$$h + \Omega(I_3 - I_2) = \Omega(I_{3e} - I_2) = \alpha_1 \tag{E10}$$

$$h + \Omega(I_3 - I_1) = \Omega(I_{3e} - I_1) = \alpha_2 \tag{E11}$$

Using this notation and substituting for the τ_i from (E6) we obtain

$$\delta\dot\omega_1 = -\frac{KB^2}{I_1}\delta\omega_1 - \frac{\alpha_1}{I_1}\delta\omega_2 + \frac{KB\Omega\delta B_1}{I_1} + 0\delta B_2$$

$$\delta\dot\omega_2 = \frac{\alpha_2}{I_2}\delta\dot\omega_1 - \frac{KB^2}{I_2}\delta\omega_2 + 0\delta B_1 + \frac{KB\Omega\delta B_2}{I_2} \tag{E12}$$

$$\delta\dot B_1 = +0\delta\omega_1 - B\delta\omega_2 + 0\delta B_1 + \Omega\delta B_2$$

$$\delta\dot B_2 = B\delta\omega_1 + 0\delta\omega_2 - \Omega\delta B_1 + 0\delta B_2$$

The last two equations are from (E3). Note that the last equations in (E3) and (E7) immediately yield

$$\delta\dot{\tilde B}_3 = 0, \quad \delta B_3 = const = 0, \quad B_3 = B \tag{E13}$$

$$\delta\dot\omega_3 = 0, \quad \delta\omega_3 = const = 0, \quad \omega_3 = \Omega$$

Equations (E12) constitute four first order ordinary linear differential equations in four unknowns. They are of the form

$$\dot{\bar X} = A \bar X \tag{E12,a}$$

where A is a square matrix with constant coefficients.

Now, if A is negative (positive) definite it may be immediately shown that the system described by (E12,a) is assymptotically stable (unstable) by applying Liapunov's (Chetayev's) stability theorem to the function $v = \bar{x} \cdot \bar{x}$. Unfortunately, for our problem, as for many others, A is sign indefinite and the theorems noted are not so easily applied.

Instead, we Laplace transform (E12,a)

$$s\bar{X}(s) - \bar{x}_0 = A\bar{X}(s) \qquad (E14)$$

Thus

$$\bar{X}(s) = (A - Is)^{-1}(-\bar{x}_0) \qquad (E15)$$

and

$$\bar{x}(t) = L^{-1}\{(A - Is)^{-1}(-\bar{x}_0)\} \qquad (E16)$$

Note that, as shown by E15, all the components of $\bar{X}(s)$ (remember, $\bar{x}(t)$ and $\bar{X}(s)$ are column vectors) have a denominator $D(s)$ and

$$D(s) = \text{Determinant}(A - Is) \qquad (E17)$$

Now, $D(s)$ may also be written as

$$D(s) = (s - s_1)(s - s_2)\cdots(s - s_n) \qquad (E18)$$

where the s_i are the roots of $D(s) = 0$ or $\text{Det}(A - Is) = 0$. Alternately, the s_i are the eigenvalues of the matrix A. Obviously, for stability, it is necessary and sufficient that the real parts of the s_i be less than zero.

To determine the s_i we expand

$$D(s) = \text{Det}(A - Is) = 0 \qquad (E19)$$

obtaining the (linearized system) characteristic equation in terms of the various system parameters. The matrix A from (E12) is

$$A = \begin{bmatrix} -KB^2/I_1 & -\alpha_1/I_1 & KB\Omega/I_1 & 0 \\ \alpha_2/I_2 & -KB^2/I_2 & 0 & KB\Omega/I_2 \\ 0 & -B & 0 & \Omega \\ B & 0 & -\Omega & 0 \end{bmatrix} \qquad (E20)$$

and the expansion (E19) yields, after a little algebra

$$I_1 I_2 s^4 + (I_1 + I_2)KB^2 s^3 + (I_1 I_2 \Omega^2 + \alpha_1 \alpha_2 + K^2 B^4)s^2$$
$$+ KB^2\Omega(\alpha_1 + \alpha_2)s + \alpha_1\alpha_2\Omega^2 = 0 \qquad (E21)$$

We require, for stability, that all the solutions of (E21) lie in the left half-plane. We may attack (E21) either numerically or analytically. Consider the analytical approach first.

If we apply Routh's Criterion to (E21) we obtain, as the necessary and sufficient conditions for stability, that the following quantities must be greater than zero if a_1 is

$$I_1 I_2 > 0 \qquad\qquad\qquad (a1)$$

$$(I_1 + I_2)KB^2 > 0 \qquad\qquad\qquad (a2)$$

$$\phi_1 = I_1 I_2 \Omega^2 + \alpha_1 \alpha_2 + K^2 B^4 - \frac{I_1 I_2 \Omega (\alpha_1 + \alpha_2)}{I_1 + I_2} > 0 \qquad (a3)$$

$$\phi_2 = KB^2 \Omega (\alpha_1 + \alpha_2) - KB^2 \Omega^2 \alpha_1 \alpha_2 (I_1 + I_2)/\phi_1 > 0 \qquad (a4)$$

$$\alpha_1 \alpha_2 \Omega^2 > 0 \qquad\qquad\qquad (a5)$$

Requirement (a1) is trivial, and so is (a2) as long as K>0. (a5) is satisfied as long as the body is dynamically stable (see appendix C) without the control system. We must address ourselves to (a3) and (a4). We first note, en passant, that if we set K = 0 in (E21) we obtain the characteristic equation for the system without control torques. This situation was examined in appendix C. Equation (E21) becomes

$$s^4 + as^2 + b = 0 \qquad\qquad\qquad (E22)$$

where

$$b = \frac{\alpha_1 \alpha_2 \Omega^2}{I_1 I_2}$$

and

$$a = \Omega^2 + \frac{\alpha_1 \alpha_2}{I_1 I_2} + K^2 B^4$$

It is trivial to show that the requirements for stability of (E22) are that a>0, b>0. Obviously b>0 is the stronger condition; we obtain the requirement

$$\alpha_1 \alpha_2 > 0 \qquad\qquad\qquad (E23)$$

and this is always satisfied if the system is stable without the control system.

We return now to conditions (a3) and (a4). ϕ_1 can always be made greater than zero for sufficiently large gain K. For K = 0, (a3) may be written as

$$2(I_1 + I_2) + \frac{(I_1 + I_2)(I_{3e} - I_2)(I_{3e} - I_1)}{I_1 I_2} > 2I_{3e} \qquad (E24)$$

Various cases of this can now be considered. If Ω is small and less than zero (opposite in sense to $\bar{h}$), (E24) is easily satisfied. If Ω is small and positive this relation is always satisfied. For large values of Ω (either sign) $I_{3e} \rightarrow I_3$. If I_3 is a major or minor axis (E24) is always satisfied. If I_3 is an intermediate axis, (E24) may fail (for K = 0) for sufficiently large Ω (which is required to make $I_{3e} \rightarrow I_3$). Before this happens the criterion (a5) will fail, so we need not concern ourselves with (a3) except perhaps for some intermediate values of Ω, such that neither $I_{3e} \gg I_1, I_2$ nor $I_{3e} \rightarrow I_3$. If we make some simplifying assumptions- namely $I_1 = I_2 = I$, and $I_{3e} = nI$, (E24) reduces to

$$(n - 1)^2 > (n - 2) \qquad (E25)$$

which holds for all values of n, positive and negative. It would seem that except for a rather oddly configured satellite that $\phi_1 > 0$, and (a3) presents no problems. In fact, unless the ratio between I_1 and I_2 is greater than 5:1, a3 must be satisfied. Additionally, for K > 0, ϕ_1 increases and (a3) presents even fewer problems. On the basis of this analysis we turn our attention to condition (a4).

$$\phi_2 = KB^2\Omega(\alpha_1 + \alpha_2) - KB^2\Omega^2\alpha_1\alpha_2(I_1 + I_2)/\phi_1 > 0 \qquad (a4)$$

$$\phi_2 = KB^2\Omega^2[(2I_{3e}-I_1-I_2)-\Omega^2(I_1+I_2)(I_{3e}-I_1)(I_{3e}-I_2)/\phi_1] > 0 \qquad (E26)$$

Now, from condition (a5) we see that the second term of (E26) is less than zero. Thus, if the first term is less than zero, this system is unstable. It is a requirement then, for stability, that I_{3e} be the major principal axis of the satellite (ie, $I_{3e} > \max(I_1, I_2)$).

Now, for our system, I_3 is a minor axis, and for large values of Ω, $I_{3e} \rightarrow I_3$, so the system is unstable. For $\Omega > 0$, in fact, I_{3e} is always a minor axis (if I_3 is). The only stable values for operation are positive values not "large". In examining this problem we now turn to a numerical study.

For a numerical evaluation, the following values are substituted in conditions (a3), (a4) and (a5).

$I_1 = 35$ slug-ft^2 (47.5 Kg-m^2)

$I_2 = 50$ slug-ft^2 (67.9 Kg-m^2)

$I_3 = 25$ slug-ft^2 (33.9 Kg-m^2)

B = 0.5 Gauss

h = 0.7 lbf-ft-sec (0.944 Nt-m-sec)

K = 10^4 pole-cm/(5 x 10^{-4} Gauss/sec) = 2×10^{-8} Nt-m/(Tesla^2/sec)

K is chosen by calculating the describing function for a flip-flop control law and a system oscillation amplitude of about ω_0. Note that K decreases as ω grows and hence the system becomes less stable. Now, (a3) becomes

$$\phi_1 = 5019\Omega^2 - 97.7\Omega + 0.89 + K^2B^4 > 0 \qquad (E27)$$

(E27) has no real zeros even for the worst case (K = 0), so we need not concern ourselves with it further. We next look at (a5) and determine the maximum value of Ω for which I_{3e} is a major axis (recalling that this is required by ϕ_2)

$$\frac{h}{\Omega} + I_3 > \max(I_1, I_2) \qquad (E28)$$

The result is $\Omega_{max} = 0.0278$ rad/sec.

Finally, we examine ϕ_2 (expression a4):

$$\frac{\phi_2}{KB^2} = -\frac{53315\Omega^4}{\phi_1} + \frac{5185\Omega^3}{\phi_1} - (47.6 + \frac{102.7}{\phi_1})\Omega^2 + 1.89\Omega \qquad (E29)$$

We are interested in determining whether $\phi_2 > 0$ for $.0278 > \Omega > 0$. This is required if we are to have any stable region of operation at all. This task is somewhat simplified by first examining ϕ_1. We see that ϕ_1 is relatively constant for this range of Ω. In fact, for K = 0,

$$(\phi_1 = 5015\Omega^2 - 97.7\Omega + 0.89) \qquad (E30)$$

$$\phi_1(0) = 0.89$$

$$\phi_1(.01) = 0.515$$

$$\phi_1(.02) = 0.94$$

$$\phi_1(.0278) = 2.05$$

and
$$\phi_{1min} = 0.418 \text{ at } \Omega = 0.00973$$

K = 0 is surely the worst possible case, since as K increases, so does ϕ_1 and thus ϕ_2 will increase if we are in the range $0 < \Omega < 0.0278$.
Now, (E29) has only two real roots. One is obviously at $\Omega = 0$, the other lies at about $\Omega = 0.05$. Between these roots $\phi_2 > 0$. Thus we have ensured a stable region of operation for

$$0.0278 > \Omega > 0.0 \qquad (E31)$$

One may convince oneself of the truth of this result by returning to equation (E21), substituting values for the coefficients and then solving the resulting equation numerically for the roots s_i. It is, of course, a necessary and sufficient condition for stability that all the s_i lie in the left half plane.

Substituting into (E21) we obtain

$$3225s^4 + 57.7s^3 + (3687\Omega^2 - 45\Omega + 1.14)s^2 + (-23.8\Omega^2 + 0.944\Omega)s$$

$$+ (462\Omega^4 - 45\Omega^3 + 0.89\Omega^2) = 0 \qquad (E32)$$

Numerical solutions for the roots of (E32) confirm the results (E31) obtained thru the Routhian analysis.

All of the results obtained in this appendix, up to this point, are only valid in a rigorous sense, for motion in the "neighborhood" of the equilibrium point. Otherwise the linearizations made in obtaining the system equations (E12) are not allowable. Nonetheless, considerable digital simulation experience indicates that the results obtained in this section are in fact applicable to the real (non-linear) system. Let us consider then the significance of these results. First of all, they indicate that the terminal attitude of the satellite must be such that the momentum bias points along the orbit normal, rather than opposite it. We remember that in the terminal condition the satellite has an angular velocity $2\omega_o e_n$. If the satellite were to acquire "backwards", then the angular velocity $2\omega_o e_n$ would correspond to $\Omega < 0$, and we have seen that this equilibrium point is an unstable one. We have seen that there are no stable equilibrium points with very high angular velocities; the greatest being $\Omega = 0.0278$ rad/sec. This means that, with the exception of possible limit cycle difficulties, this system should be well behaved and the control theory developed in the main body of this report provides for the positive despin of the vehicle. The author also wishes to note that he has observed no limit cycle behavior during extensive digital simulation of this system.

The reader is probably asking, at this point, "How about the equilibrium points II and III enumerated in Appendix D". The answer is that the author has not yet had time to perform a rigorous stability analysis for these points. However, on the basis of digital simulation it appears that they are not stable points unless $\dot{\omega}_1 = \omega_2 = 0$, which is of course the case treated here.

Finally, we note that even the stable equilibrium states determined here present no real despin problems. The investigations here dealt, for simplicity, with a fixed external magnetic field. In fact, even in the worst case (an Equatorial orbit with the pitch wheel along the orbit normal) the Earth's field appears to cone about with a half angle of 11.7°. If the satellite is to remain in the equilibrium state, it too must cone about the orbit normal at frequency ω_o. The control torques maintaining the equilibrium state (i.e. damping out disturbances - which is how the coning of the $\bar{B}$ field appears to the system) are essentially dissipative in nature, since they oppose sensed velocities. The final result is then that even the small residual spin $\Omega < .0278$ rad/sec along the momentum bias will be eliminated due to this previously unconsidered effect.

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MAGNETOMETER NOISE MEASUREMENTS

PURPOSE OF THE STUDY

This study is part of an attempt to detect apparent changes in the earth magnetic field from a satellite during initial attitude acquisition. A system designed to accomplish this (Figure 1) consists of the Schonstedt RAM-5C-NB Magnetometer followed by a differentiator circuit. The output of the differentiator would be fed to a threshold detector of an attitude control system. This report evaluates the noise type and levels present in the magnetometer-differentiator combination.

SENSOR SYSTEM
FIGURE 1

GENERAL OPERATION OF THE MAGNETOMETER-DIFFERENTIATOR

The proposed magnetic aspect system (Figure 1) is composed of a probe and associated electronics (Magnetic Aspect Sensor) purchased from Schonstedt, and a differentiator circuit. The sensor provides a "d.c." output voltage proportional to the component of the magnetic field coaxial with the probe. The differentiator circuit not only differentiates, but provides attenuation over a range of frequencies outside of the range of vehicle rotation frequencies. Primarily, two circuits were used during the study, one with a corner frequency W_C of 1 radian per second and the other with a corner frequency of 5 radians per second. The approximate equivalent circuits are shown in Figure 2.

DIFFERENTIATOR WITH $W_C = 1$

FIGURE 2a

DIFFERENTIATOR WITH $W_C = 5$

FIGURE 2b

FIGURE 3.

~10 DEGREE PHASE SHIFT FROM IDEAL DIFFERENTIATOR

10 MΩ

.02 µf

20 KΩ 10 µf

$$Y(j\omega) = \frac{100W}{1 + \frac{W^2}{25}}$$

W ORBIT RATE

MODEL

DATE

TRANSFER FUNCTION FOR DIFFERENTIATING THE APPARENT EARTH MAGNETIC FIELD

W RADIANS/SEC.

FIGURE 3b

The mathematical representation is

$$Y(jW) = \frac{-j100W}{(1+j\frac{W}{W_c})^2} \quad ; \text{ W is in radians per second}$$

Figures 3a and 3b are the Bode plots corresponding to the circuits of Figures 2a and 2b. Zero db is defined equivalent to unity gain.

TEST RESULTS

Figure 4 is the actual circuit configuration of the differentiator with 1 radian/sec. corner frequency. Separate FET's were used. The circuit with a corner frequency of 5 radians/sec. employed the 2N3922 dual FET's.

DIFFERENTIATOR SCHEMATIC
(Wc = 1)   FIGURE 4

Three magnetometers SN3881, 3882, and 3883 were used at various times during the testing; but for consistency, 3883 was subjected to each test.

The outputs of the $W_c=1$ and $W_c=5$ circuits were measured when these circuits were used in combination with magnetometer 3883. The two tests were not run simultaneously. The magnetometer was centered radially in a wooden cylinder which in turn was centered radially in an iron pipe. The pipe was then closed. A magnet external to the pipe supplied the magnetic field. The magnet produced a 2.3 volt magnetometer output. The output of the differentiators were recorded using a Brush recorder.

The peak to peak values obtained are

8mV for the W_C = 1 circuit,

80mV for the W_C = 5 circuit.

Figures 5a and 5b are samples of the recordings from which the values were obtained.

To determine the noise generated by the differentiator circuit, the output was measured with the input shorted. Both circuits were placed in a metal box and the outputs simultaneously measured using the Brush recorder. The peak to peak output was half a millivolt for the W_C = 1 and 2 millivolts for the W_C = 5 circuit.

The magnetometers themselves do not have identical noise characteristics. Figure 6 is a Brush recording showing the noise output from the W_C = 5 circuit fed by each of the three magnetometers. Each of the magnetometers was tested in the pipe with the magnet supplying the same field in each case. A considerable difference between the noise levels of the magnetometers is apparent. Disturbances, some deliberately created, in the laboratory verify that the magnetometers and the differentiator were functioning during the measurements.

Two magnetometers, each in combination with a W_C = 1 differentiator were tested simultaneously to show that spikes which were being observed on the oscioolgraph are not internally generated. Oscillographs obtained during the night hours also tend to confirm that the largest spikes are not internally generated. The night graphs are almost without the large spikes which are common in the data taken during normal working hours.

Oscillographs were run for 1 hour with the W_C = 1 differentiator and then for 1 hour with the W_C = 5 differentiator. These recordings show that noise with a period on the order of minutes is negligible by comparison with the amplitude of the noise envelope.

Root-mean-square (RMS) measurements were obtained to indicate the general level of the noise as a function of W_C. The measurements were obtained graphically from the Brush oscillographs. Three circuits were used in turn with magnetometer 3883. The magnetometer was enclosed in the pipe and the field supplied by a magnet external to the pipe. All three differentiators were of the same mathematical form, all had K = 100, and all had the same input capacitor and output resistor. With those restrictions, the noise levels may be considered to be a function of W_C. The equivalent circuits for two of the differentiators are Figures 2a and 2b, the third (W_C = 0.5) is shown by Figure 7.

Vertical Scale: 1mV/small div.
Horizontal Scale: 5mm/sec.

Noise Output of Differentiator with $W_C = 1$ rad/sec. in
Combination with Magnetometer SN 3883.

FIGURE 5A

Vertical Scale: 5mV/small div.
Horizontal Scale: 5mm/sec.

Noise Output of Differentiator with $W_C = 5$ rad/sec. in
Combination with Magnetometer SN 3883.

FIGURE 5B

3882   3881   3883

Comparison of Noise Levels from Three Magnetometers.
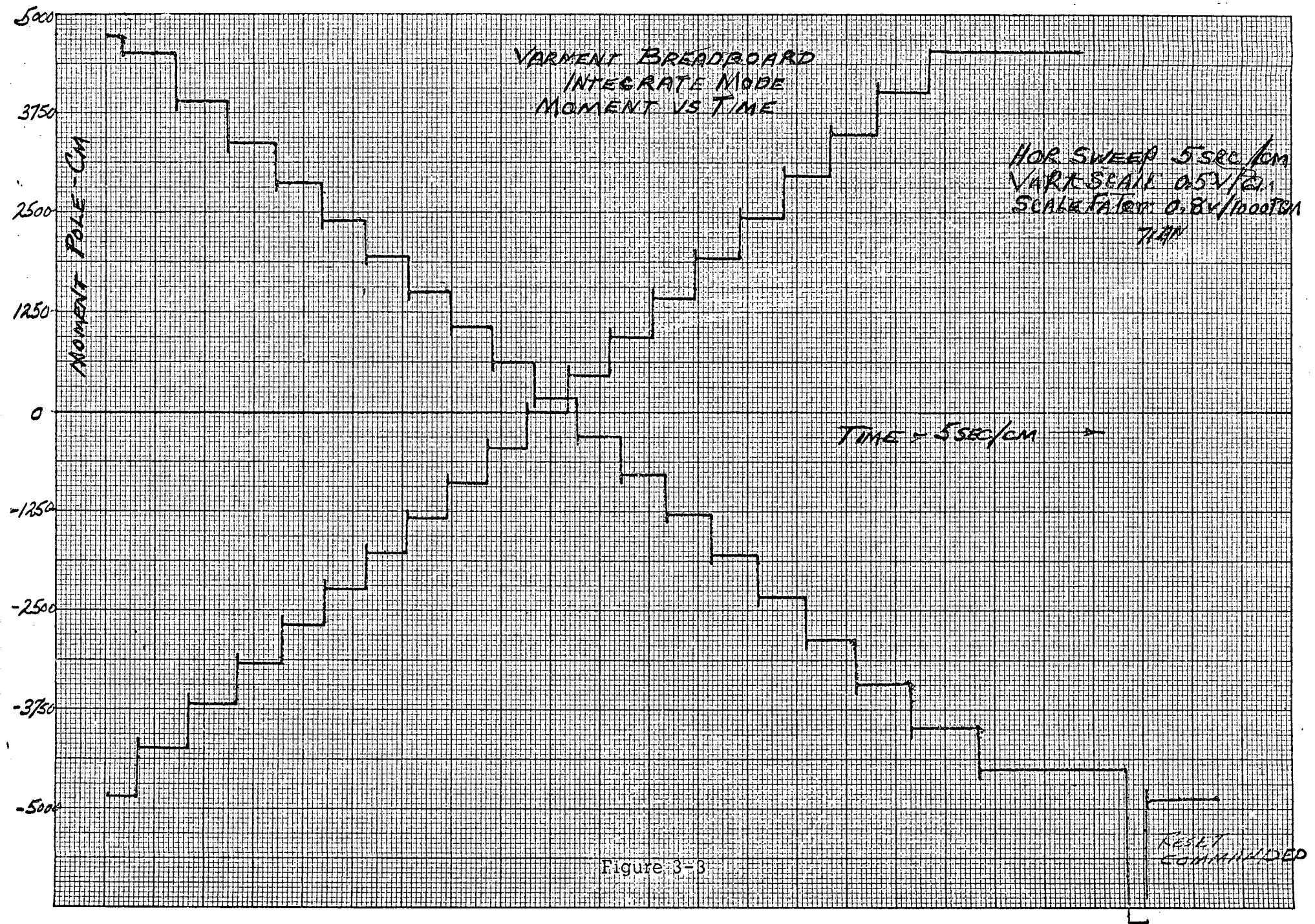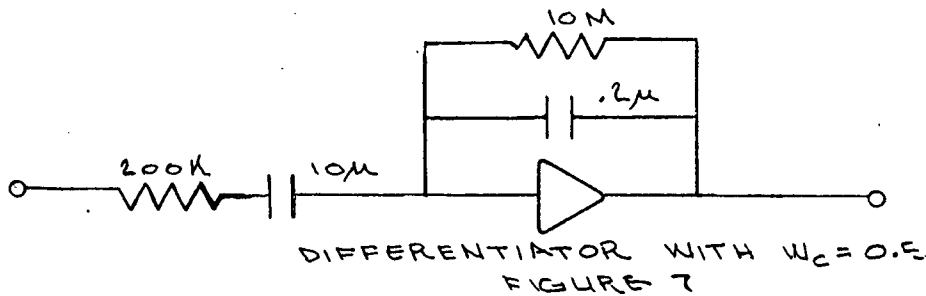Output of Differentiator with $W_C = 5$ rad/sec.

FIGURE 6

Figure 3-3

DIFFERENTIATOR WITH $W_c = 0.5$

FIGURE 7

The results of the graphical analysis are given in Table 1.

W_c radian/sec.	Magnetometer 3883 Output Volts d.c.	Differentiator Output Noise Millivolts RMS
0.5	1.6	0.6
1.0	2.3	1.3
5.0	2.3	8.1

TABLE 1

To obtain the RMS voltage graphically, the instantaneous value of the noise voltage is measured at 1 second intervals. The square root of the mean of the square of these measurements is the RMS voltage.

MAGNETOMETER OUTPUT CHARACTERISTICS

Tests show that the d.c. output impedance of the magnetometer is 3.6K ohms. The 20K ohm resistance shown in Figure 2b actually includes the 3.6K ohm output impedance.

At a frequency of $W_c = 0.5$ radian/sec., the impedance of the 10µ farad differentiator input capacitor is 200K ohms, so there is neglibible loading from d.c. to $W = 0.5$ radian/sec.

Discussions with the vendor indicate that no electrical isolation problems exist in connection with the output of the magnetometers.

CONCLUSIONS

To get some idea of the feasibility of using the particular magnetometer-differentiator combination studied, some assumptions of physical conditions will be made.

For purposes of this study, a satellite will be assumed in a 500 mile high circular orbit. The RAM-5C magnetometer has a sensitivity of 4 volts per oersted. Assuming a maximum sea level field strength of 580 milliguass in the orbit plane, and an inverse cube variation of field strength with orbit radius; a voltage of $\approx$ 1.6 volts maximum would be present at 500 miles.

The Bode plots, Figure 3, give the voltage gain of the corresponding differentiators as a function of the frequency of a sinusoidal input. For simplicity, assume the satellite to be rotating about an axis which is fixed relative to the earth. Assume we are interested in rotation frequencies from 0.5 radians/sec. down to the orbit rate, and assume the orbit frequency is .001 radian/sec.

According to the Bode plots, a magnetometer signal at the orbit frequency would be attenuated by a minimum of 20db. For the 500 mile orbit, the peak differentiator output would be a maximum of ±160mV. If the angle between the magnetometer probe and the orbit plane were such as to reduce the field strength by a factor of 0.1, then the signal would be attenuated another 20db and the peak output of the differentiator would be ±16mV.

Suppose threshold detectors to be at the output of the differentiator. Based on the above discussion, the threshold levels might be set between ±16mV and ±160mV. Recall that the peak to peak noise values recorded on the oscillograph were

8mV for the W_c = 1 circuit,
80mV for the W_c = 5 circuit.

The oscillographs show that the noise level measured at the output of the differentiator is dependent on the d.c. output voltage of the magnetometer. The reason is not known.

The study does not conclusively define the limitations of this particular magnetometer-differentiator combination. However, based on the measurements, the prospects for this combination appear promising.

MAGNETOMETER NOISE MEASUREMENTS

Addendum to Report No. 90293, Aug. 27, 1971

Report No. 90293 gives the results of noise measurements for the Schonstedt RAM-5C-NB Magnetometer in combination with a differentiator circuit. As mentioned in that report, the noise level measured at the output of the differentiator is dependent on the d.c. output voltage of the magnetometer.

Oscillographs of the noise at the output of the magnetometer itself shows a dependence on the d.c. output voltage of the magnetometer. To obtain this more recent data, magnetometer 3883 was enclosed in the iron pipe and a constant magnetic field supplied by a magnet external to the pipe. The output of the magnetometer was coupled to the preamp of the Brush Recorder through a 10µf capacitor. Samples of these recordings are presented in the Figure.

A close look at the oscillographs show the noise level to be higher for a magnetometer output voltage of -2.4V d.c. than for an output voltage of +2.4V d.c. Since all of the data in Report No. 90293 was obtained for positive output voltages, the following question arises. If data were taken for negative and positive instead of just positive output voltages, might magnetometer 3881 be found to have greater p-p noise than 3883, instead of the other way around. To resolve this doubt, 3881 was tested in combination with the W_C = 5 differentiator; the magnetometer was enclosed in the pipe and the field supplied by a magnet external to the pipe.

The following table shows the p-p differentiator output noise as a function of magnetometer output voltage. The data is only approximate due to the difficulty of defining p-p noise. However, this data clearly shows that the noise levels are well below the 80mV p-p measured with 3883.

Differentiator Output Noise vs Magnetometer 3881 DC Output

Magnetometer Output Volts DC	W_C=5 Differentiator Noise Output Millivolts p-p
-2.4	25
-2.0	25
-1.0	14
0.0	9
+1.0	20
+2.0	19
+2.4	35

CONCLUSIONS

   Noise levels vary from magnetometer to magnetometer. Noise levels also vary with field strength. Finally, and most important, if rates on the order of orbit rates are to be detected, the results of the tests with the $W_C = 5$ differentiator are inconclusive. Simulation will be needed to provide the answer. However, if a relay were installed to convert the $W_C = 5$ differentiator to a $W_C = 1$ differentiator during acquisition, detection at orbit rates should be feasible.

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vdc = +2.4

vdc = -2.4

vdc = +1.5

vdc = 0.0

vdc = +1.0

vdc = +0.5

horizontal scales:
        20mm/sec.
vertical scales:
        1mV/small division

Output Noise Voltage for Various DC Output Voltages for
Magnetometer S/N 3883

III Electronics for Control of Variable Permanent Magnets

1.0 Introduction

2.0 Control Electronics

3.0 Variable Permanent Magnet Breadboard Controller

III Electronics For Control of Variable Permanent Magnets

1.0 Introduction

In this section the circuitry is described for controlling the state of a variable permanent magnet as a linear function of an input variable. The input variable can be either a control voltage, (for control applications) or a time interval (for magnetic balancing applications).

The control circuitry consists of a Hall sensor that senses the state of the magnet, and in a feedback loop compares it with the input voltage. The high gain feedback loop forces the sensed moment to agree with the input voltage. Small deadbands are used so that once the correct moment is achieved the magnet drive is automatically turned off. The circuitry and accuracy of this method are the subject of this section.
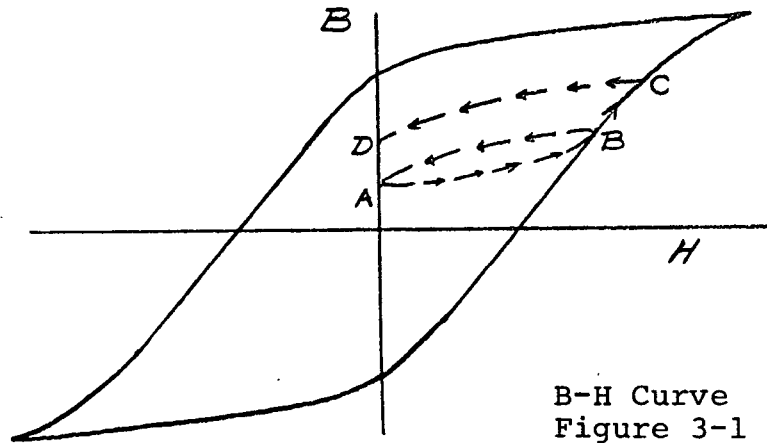
B-H Curve
Figure 3-1

The B-H curve in Figure 3-1 illustrates the inter-relationship between a permanent bar magnet and an external field.

In changing the permanent induction of the bar from A to D the path ABCD must be traversed. An external H field corresponding to point C must be applied and when removed the induction level in the bar will relax to point D. The continuous application of an external field less than that associated with point B, once the magnet has been cyclically magnetized, will not alter the permanent induction. The minor loop AB is designated the recoil energy and the slope the recoil slope.

The average slope of the recoil permeability is constant for a cyclically magnetized bar up to approximately 80% of the saturated induction level and thereafter extremely non-linear.

For a bar magnet wound with a suitable coil to permit varying the induction level it is possible to predict the residual induction which will result after removing the magnetizing current. In the Varment system this is accomplished by sensing the magnetizing coil current and scaling it to cancel the recoil induction as measured by the Hall sensor.

2.0 Control Electronics

The Control Electronics for setting the state of the variable permanent magnet consists of a bipolar power bridge circuit, which supplies the necessary current with the proper polarity to the magnet coil, operated by one of two threshold circuits. See the Block Diagram (Figure 3-2).

Thresholding is achieved by two separate modes of operation designated Analog and Integrate Mode. In either mode the magnet state is continuously monitored by a Hall sensor and the current in the magnetizing coil is sensed for both magnitude and direction. The Analog Mode, which is the most straight forward, permits setting the state of the magnet in reference to an input analog voltage.

The analog input, Hall output and the current correction voltage are summed, properly scaled, and the output of this summing amplifier operates the threshold circuits which have built in hysteresis.

The amount of hysteresis does not affect the resolution but does limit the minimum incremental moment that may be obtained. Thus far the obtainable resolution has been found to be 2% of the maximum moment if the final desired moment is approached from the same direction each time. For random approaches the resolution has been found to be approximately 4%. Therefore in order to assure that the desired commanded moment will yield a differential output between the Hall voltage, the analog input and the recoil correction that is within the deadband of the thresholding circuit it is necessary to insert hysteresis

greater than 4% of the maximum capability. Otherwise
the system could continuously switch the current on and
off in a futile attempt to make the Hall voltage agree
with the commanded analog voltage.

In the integrate mode the Hall voltage is summed with a
D.C. voltage utilizing an operational amplifier with
several seconds lag. It is only necessary that the D.C.
voltage exceed the threshold to operate the magnet but
the amplitude and time lag determine the ultimate accuracy
with which any desired moment can be attained.

Upon commanding the system to the integrate mode and
energizing the magnet driver the magnet state is auto-
matically changed a fixed amount equal to the threshold
in fixed increments of time until the maximum moment is
reached. Should the desired moment fall between any
two increments it would be necessary to command a
momentary set voltage into the system that exceeds the
threshold. When removed the magnet would then be driven
to the desired state. To maintain this state it would
then be necessary to de-energize the driver prior to
the next incremental state change.

As an example consider a magnet capable of 10,000 p-cm
initially at a 1,000 p-cm state with a desired new state
of 7300 p-cm. Let the threshold be set at 500 p-cm and
the integrator time constant and input set so that the
state is changed 500 p-cm every ten seconds. The system
would require 120 seconds to attain 7,000 p-cm and the
momentary set command would be inserted at 126 seconds
from the initial integrate command. The 7300 p-cm state
would then be maintained for the next ten seconds during
which time the driver would be de-energized.

3.0 Variable Permanent Magnet Breadboard Controller

A breadboard control system consisting of a power supply card and magnet control unit was built and trimmed to control a ten inch elliptical magnet made from P-6 material. Schematics & layouts of these two cards are included in this report.
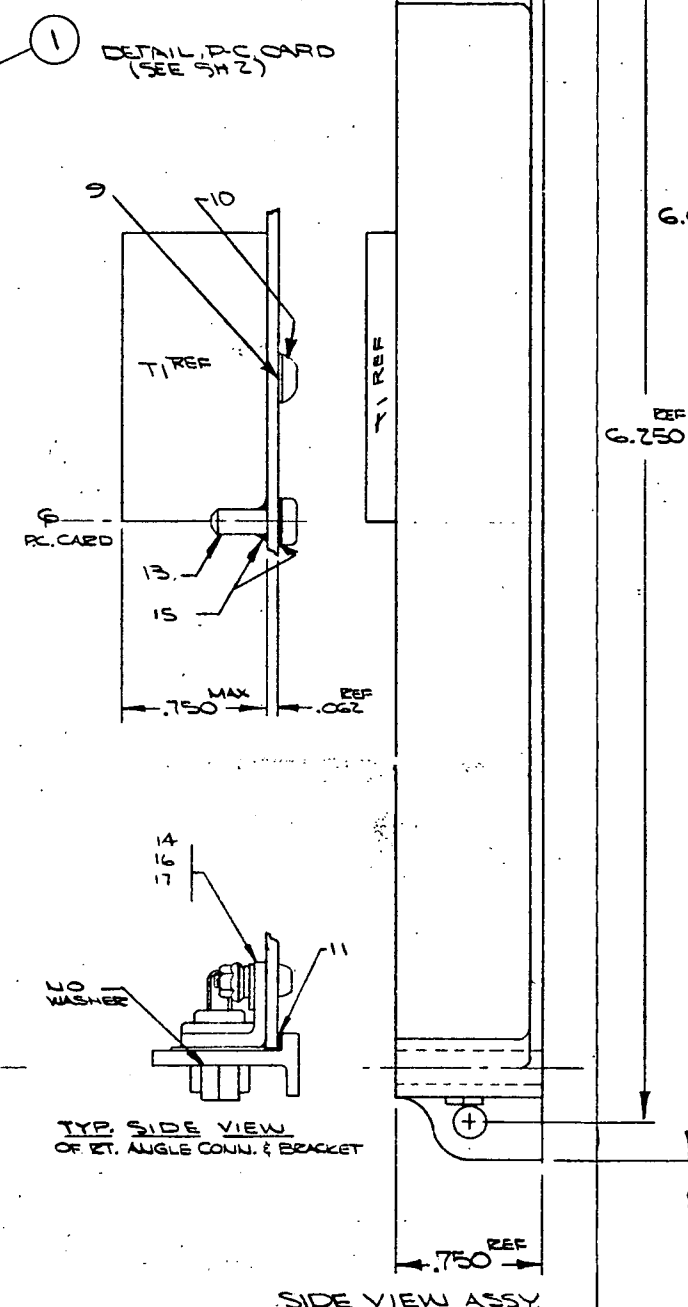
The magnet was found to have a maximum moment capability of +/- 6300 pole-cm. It was determined experimentally that utilizing the recoil current correction was satisfactory with only minor current limiting over 80% of the maximum capability of the magnet. It was also determined that attempting to reduce the threshold to better than 4% of the available moment required careful gain matching and trimming.

The X-Y plot show in Figure 3-3 show the control circuit and magnet being operated in the integrate mode starting from both the positive & negative moment to determine the linearity of the incremental moment changes. The threshold for this curve was set at $\pm$8% or approximately 400 p-cm increments.
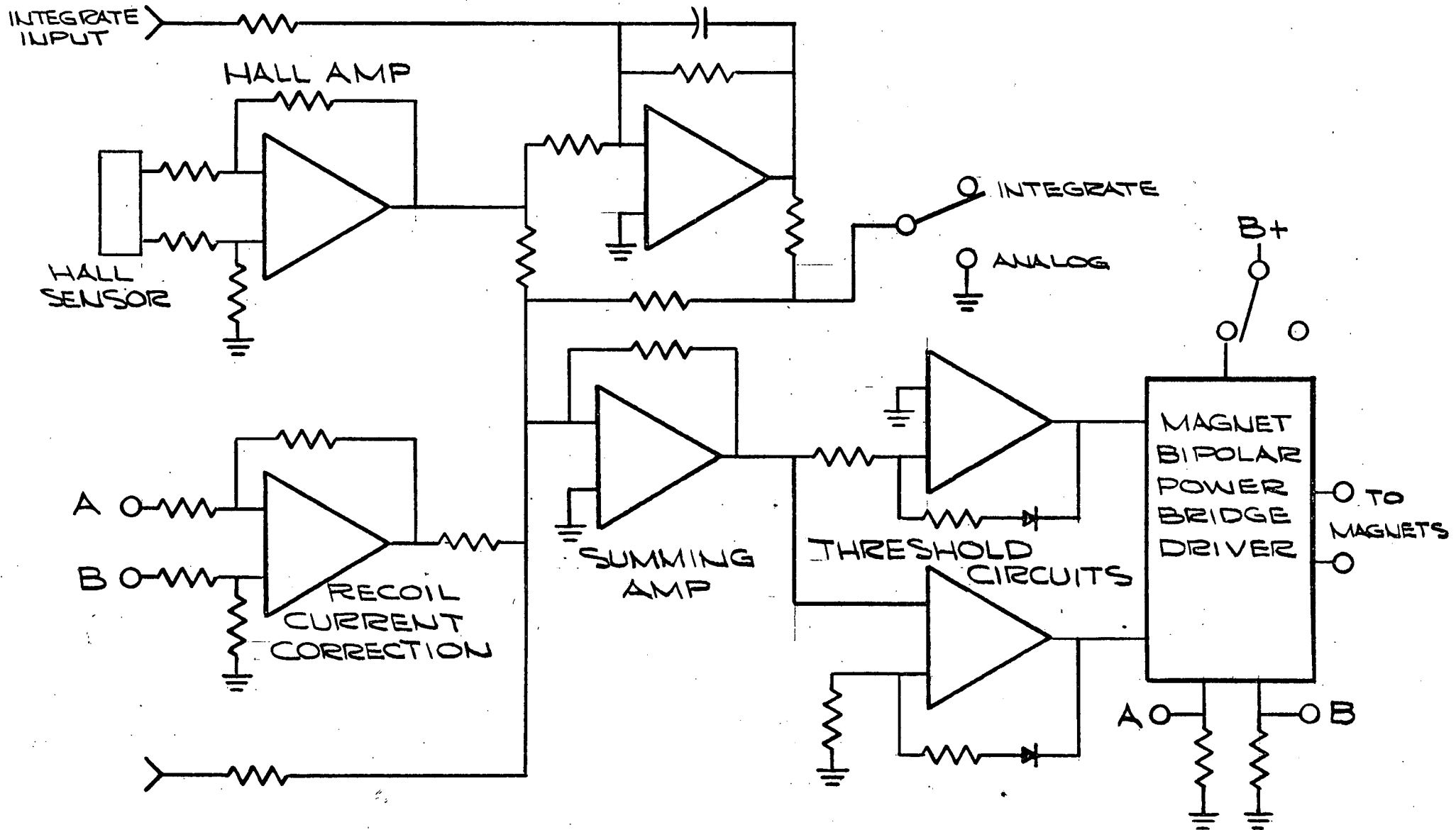
MAGNET CONTROL BLOCK DIAGRAM
FIGURE 3-2

FOLDOUT FRAME

FOLDOUT FRAME 2

5.670 REF

5.500 REF
PC CARD

5.500 REF
PC CARD

5.670 REF

CARD ℄

.500

Q4 ℄

.500

Q3 ℄

REF
.300
P.C. CARD

T 1

6.625 REF

6.250 REF

DETAIL P.C. CARD
(SEE SH 2)

TYP. SIDE VIEW
OF RT. ANGLE CONN. & BRACKET

SIDE VIEW ASSY.

TI REF

MAX
.750

REF
.062

P.C. CARD

NO
WASHER

.750 REF

G1 ASSEMBLY

NOTES:
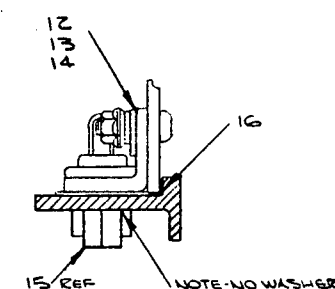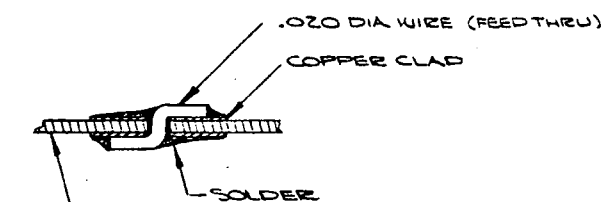
1. FORM DIODE LEADS PER A10315.

2. ASSEMBLE COMPONENTS PER MOPS 30.3.

3. SECURE LARGE COMPONENTS PER MSFC PROC. 257 PRIOR TO CONFORMAL COATING.

4. ASSEMBLE P.C. CARD TO FRAME PER MOPS 30.35.

5. ASSEMBLY TO BE CONFORMAL COATED PER MOPS 30.2, AFTER CARD TEST.

6. USE .025 DIA BUSS WIRE WITH PLASTIC SLEEVING FROM T1 TO P.C. CARD.

.020 DIA WIRE (FEED THRU)

COPPER CLAD

SOLDER

P.C. CARD

TYPICAL INTERFACIAL CONNECTION
(POINTS φ1 THRU φ13)

J1
(25P)

12

MAX
.906

T1 REF.

2.700 REF

1.350 REF

CONN

CONN

J2 (25S)

SEE SEPARATE PARTS LIST

		MATL: SEE SH 2	MODULE, POWER SUPPLY	ITHACO, INC. ITHACA, N.Y.	
		FIN: SEE NOTE 5			
		SCALE: /1	FMF: JOB# 10-2460	CODE IDENT. NO. 26502	
SK 050191	PS-1	DRAWN BY: TOSTEL NOV 10, 1971			
REFERENCE DWG	USED ON	CHECKED BY: 11-19-71	D SK-00193		
APPLICATION		APPROVAL: DFTG / ENG WAR			
		ISSUE DATE: 11-19-71	SHT OF		

UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES TOLERANCES ON
FRACTIONS ± 1/64 DECIMALS ± .010 ANGLES ± 1/2°

REVISIONS
ZONE | SYM | DESCRIPTION | DATE | APPROVAL

D SK-00193
SHT 1 OF 1

FOLDOUT FRAME 1

5.670 REF.

5.500 REF P.C. CARD

CARD ⊄

6.625 REF

6.250 REF

5.670 REF

5.500 REF P.C. CARD

.300 REF P.C. CARD

.750 REF

8   Q9 Q10   Q5 Q4   10

1 DETAIL P.C. CARD SEE SH.2

2

7

.781 REF

SIDE VIEW TYP INSTALLATION Q4, Q5, Q9, Q10
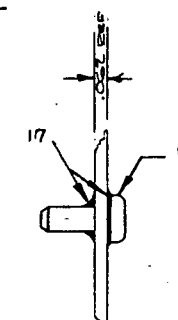
MTG CTR OF CARD

17   11

12
13
14

16

15 REF   NOTE-NO WASHER

SIDE VIEW OF RT. ANGLE CONN. & BRKT

G1 ASSEMBLY

NOTES:
1. FORM DIODE LEADS PER A10315.
2. ASSEMBLE COMPONENTS PER MOPS 30.3
3. SECURE LARGE COMPONENTS PER MSFC PROC 257 PRIOR TO CONFORMAL COATING.
4. ASSEMBLE P.C. CARD TO FRAME PER MOPS 30.35.
⚠ ASSEMBLY TO BE CONFORMAL COATED PER MOPS 30.2 AFTER CARD TEST.

.020 DIA WIRE (FEED THRU)
COPPER CLAD
SOLDER
P.C. CARD

TYPICAL INTERFACIAL CONNECTION
POINTS ⊕1 THRU ⊕23

J2 (25P)

15

⊄ CONN.   CARD ⊄   ⊄ CONN.   J1(25S)

1.350 REF

2.700 REF

SEE SEPARATE PARTS LIST

Power Supply schematic — ITHACO, Inc., Ithaca, N.Y.

REVISIONS

ZONE	SYM.	DESCRIPTION	DATE	APPROVAL

NOTES:
1. UNLESS OTHERWISE SPECIFIED:

A. - ALL CAPACITORS ARE MYLAR, GE 63F
   - VALUES IN PF'S ARE STYLE CKR.
   - POLARIZED ARE SOLID ELECTROLYTIC SPRAGUE 35OD'S.

B. - ALL DIODES ARE JAN TX IN4148
   UTX ARE UNITRODE TYPE

C. - ALL RESISTORS ARE 1/4W, 5% STYLE RCRO7.
   - * INDICATES TRIM, VALUE TO BE SELECTED AT TEST.
   - 1/10W, 1% ARE STYLE RNR55H.

D. - TRANSISTORS -
   NPN ARE JAN TX 2N2222A
   PNP ARE JAN TX 2N2907A

E. CONNECTORS
   J1 CANNON DBM 25P-K
   J2 CANNON DBM 25S-K

ELEM. DIAG.
POWER SUPPLY

ITHACO, INC.
ITHACA, N.Y.

		MAT'L:	ELEM. DIAG. POWER SUPPLY	ITHACO, INC. ITHACA, N.Y.	
		FIN:	FMF. JOB# 10-7460	CODE IDENT. NO. 26502	
		SCALE:	DRAWN BY: D. Worley OCT 4, 1971		
ASSY SK-D00193	PS-1	UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES TOLERANCES ON	CHECKED BY:	D SK-00191	
REFERENCE DWG	USED ON	FRACTIONS DECIMALS ANGLES	APPROVAL: DFTG. 11-18-71 ENG		
APPLICATION		± 1/64 ± .010 ± 1/2°	ISSUE DATE: 11-13-71	SHT 1 OF 1	

FOLDOUT FRAME 1

FOLDOUT FRAME 2

NOTES:
1. UNLESS OTHERWISE SPECIFIED:
   A. ALL CAPACITORS ARE MYLAR, GE 63F.
      VALUES IN pf ARE STYLE CKR.
      POLARIZED UNITS ARE SOLID ELECTROLYTIC SPRAGUE 350D's.
   B. DIODES ARE JAN TX IN414B.
   C. RESISTORS ARE 1/4W, 5%, STYLE RCR07.
      * INDICATES TRIM, VALUE TO BE SELECTED AT TEST.
      1/10W, 1% ARE STYLE RNR55H; 1/4W, 1% ARE PYROFILM CORP. PME 62.
   D. TRANSISTORS:
      NPN ARE JAN TX 2N2222A
      PNP ARE JAN TX 2N2907A
      ** INDICATES HEAT SINK TO FRAME.
   E. CONNECTORS:
      J1 IS CANNON DBM-25S-K
      J2 IS CANNON DBM-25P-K

ITHACO, INC.
ITHACA, N.Y.

ELEM. DIAG.

VARMENT

SK D00146

IV HORIZON SCANNER BREADBOARD

1.0 Introduction

2.0 Description of the X Scanner

3.0 Operational Description

4.0 Figures

IV HORIZON SCANNER BREADBOARD

1.0 Introduction

As a part of the work under the subject contract, a demonstration breadboard of a new proprietary horizon scanning system developed by ITHACO was fabricated. The purpose of this breadboard is to illustrate the adaptation of this "X Scanner" to an existing Bendix Reaction Wheel.

There are many solutions to solving the circular body (Earth) edge scanning problems. Complexity should be avoided.

We think this has been done in the following description of the "X Scanner". The Patent Disclosure, Appendix #1 and #2, gives a detailed description of the scanner design and detector arrangement.

The basis for this design is to keep the system to its simplest, lightest and most reliable configuration. In general, the fewer the number of parts, the less there is to get out of order.

2.0 Description of the X Scanner

This system consists of one reaction wheel which mounts the optical scanner parts. This reaction wheel mounts the scanning double dove prism so that there is only one set of bearings and one scanning optical element. The single window, one sealing joint, is mounted rigidly to the case. The lens is also rigidly mounted to the case, as is the bolometer (or detector system), so that the optics cannot move or jump in relation to each other.

The scan prism drive reaction wheel is oriented so that its axis of rotation is parallel to the flight axis, the center of the earth.

The detector system, as explained in Appendix #2, consists of three detectors in the shape of a right triangle with a flake at each apex. This is required because the double dove prism, being set at 45° or close to it (this can vary due to requirements), rotates the image at twice this angle. It was necessary to have two flakes contact the earth at the same time for Sun control or compensation.

3.0 Operational Description

The scan pattern is such that one leg of the "X" crosses the earth with 180° rotation of the reaction wheel and the other leg with the next 180° rotation of the wheel. With the prism tilted 45° to the axis, the first 180° rotation crosses the earth and then the second 180° rotation has the prism tilted 90° from the first position. In this manner the common flake and the number 1 apex flake cross tangent to the earth on the first 180° rotation, and the common flake and the number 2 apex flake cross tangent to the earth on the next 180° rotation of the prism.

In this manner one reaction wheel, one window, one prism, one lens, one detector and preamp will give a complete scan report on one revolution of the reaction wheel. The line of sight contact is tangential to the earth for the pair of flakes. Also, the line of sight of this optical system is in line with the optics, which means good optical performance (this is an improvement over some previous systems).

The mathematical analysis is given by Appendix #3 (Report #90274) and Appendix #4 (Report #90276).

4.0

Figure IV-1 shows the demonstration model and the double dove prism. The large diameter at the left simulates an 8" reaction wheel. This shows the prism without any window.

Figure IV-2 shows the synchronous type scanner with a small plain window. The prism can be quite close to the reaction wheel.
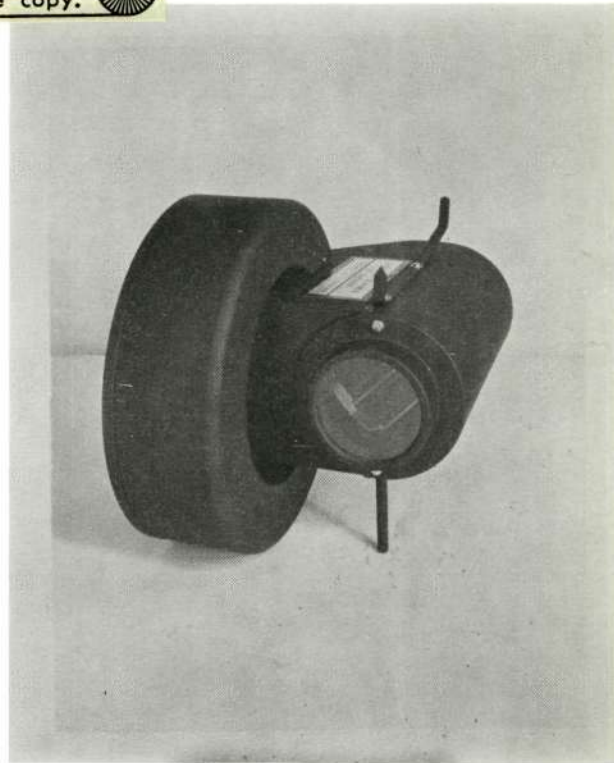
Figure IV-3 shows the elliptical orbit scanner with a dome to cover the wide angle of this scan. The large distance between the prism and reaction wheel is required for the line of sight to clear the reaction wheel case for the lower altitudes.
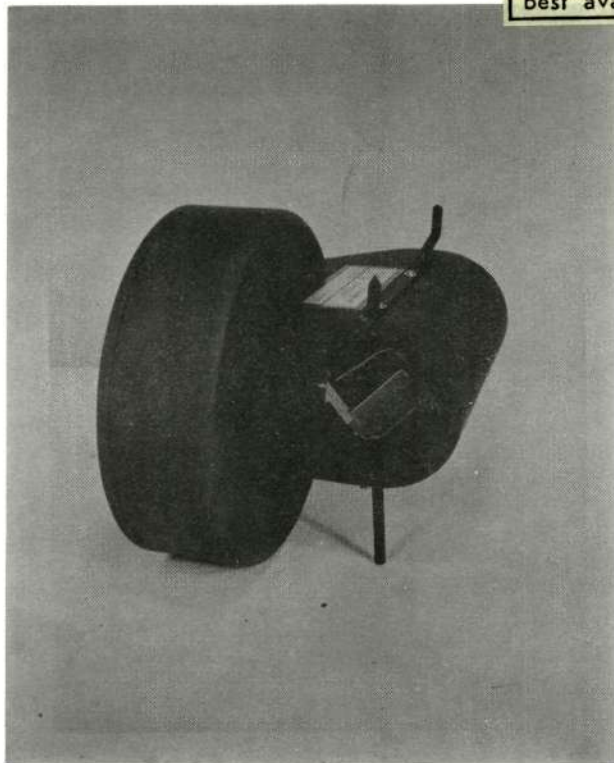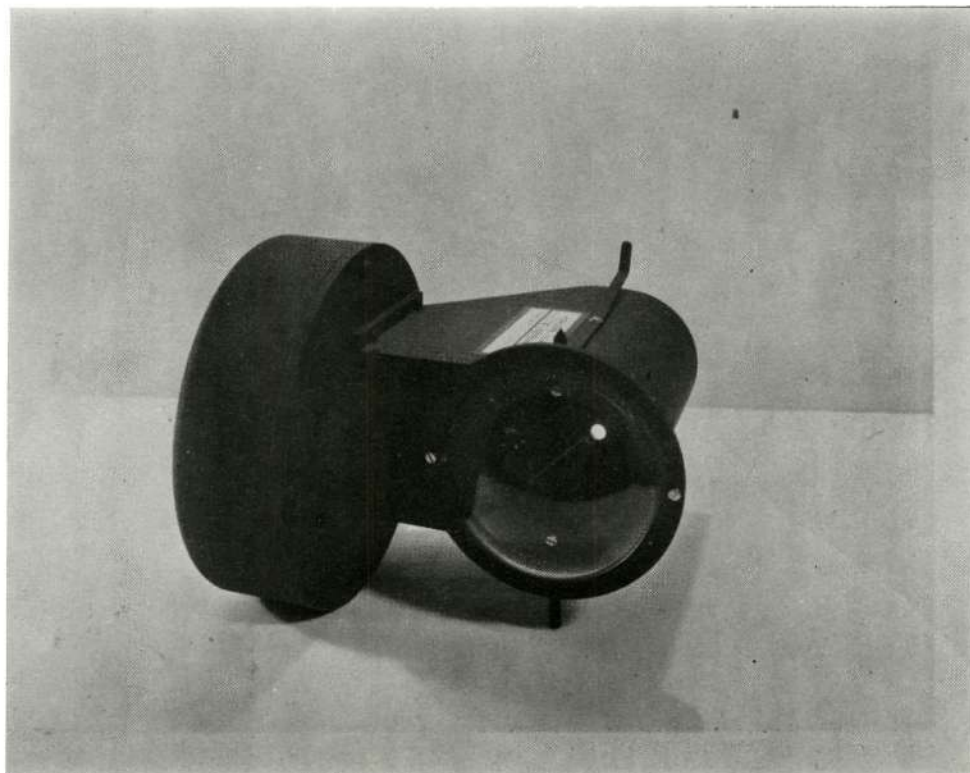
Figure IV-1

Scanner Without Window
(to show prism)

Figure IV-2

Synchronous Orbit Scanner
(with flat window)

Figure IV-3

Elliptical Orbit Scanner
(with dome)

PATENT DISCLOSURE

HORIZON SENSOR - ANY ALTITUDE SATELLITE

Purpose - This horizon sensor is to operate at any altitude to the earth from approximately 500 N. Mi. out to greater than 24,000 N. Mi. within an accuracy of 0.1 degree.
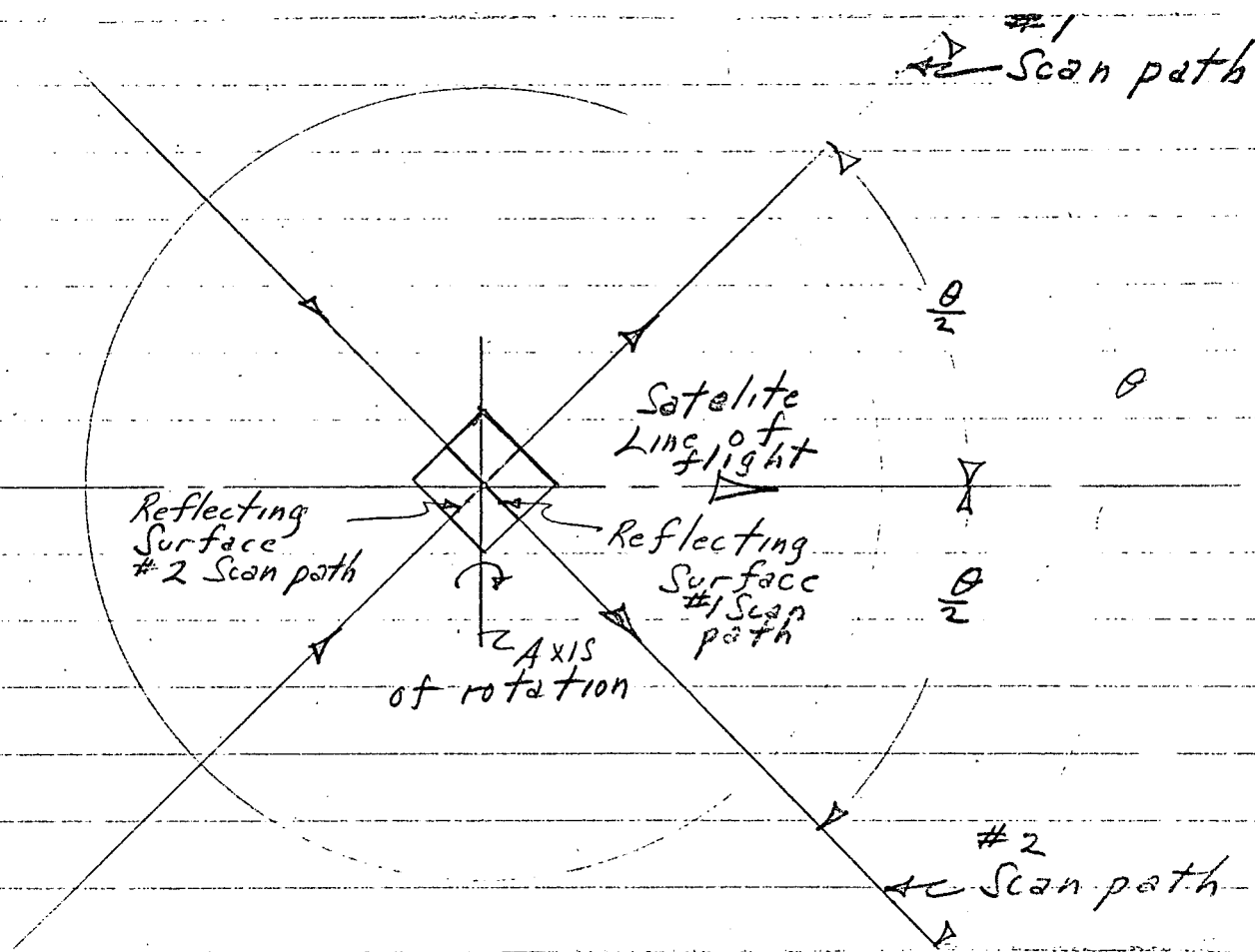

FIGURE I

Invention - This invention describes a scanner, single head consisting of a detector, a lens system and a scanning prism mounted on an axis of rotation shown basically in Figure I. This scans two paths at an angle θ to each other and at $\theta/2$ to the line of flight. The angle θ in the sketch is shown as 90° to illustrate.

This accomplishes the purpose of controlling both Pitch and Roll with one scanner (previous systems used two scanners).

Invented by
 Kennard Watson Harper   April 1, 1971   *Kennard Watson Harper*
                                               apr. 8, 1971
Witnessed and Understood by

1) *David Johnson*   4-8-71   (4-8-71 85.)
2) *E.C. Mitchell* 4-8-71

The system shown basically in Figure II shows the various parts:

shaft + prism mount

Double Dove Prism

Lens op.

Reflecting Surface

Bolometer

Plan View

Axis of rotation

Bolometer

Lens

Filter

Rotation

Reflecting Surface

Downward Look

This shows Light rays Thru prism

45° Look

22.5°

45°

Dome or Flat Window

View "A"

Scan Looking Straight down

Side Elevation

32.76°

Dome or Flat Window

View B

Scan at 45° down + To one side

Side Elevation
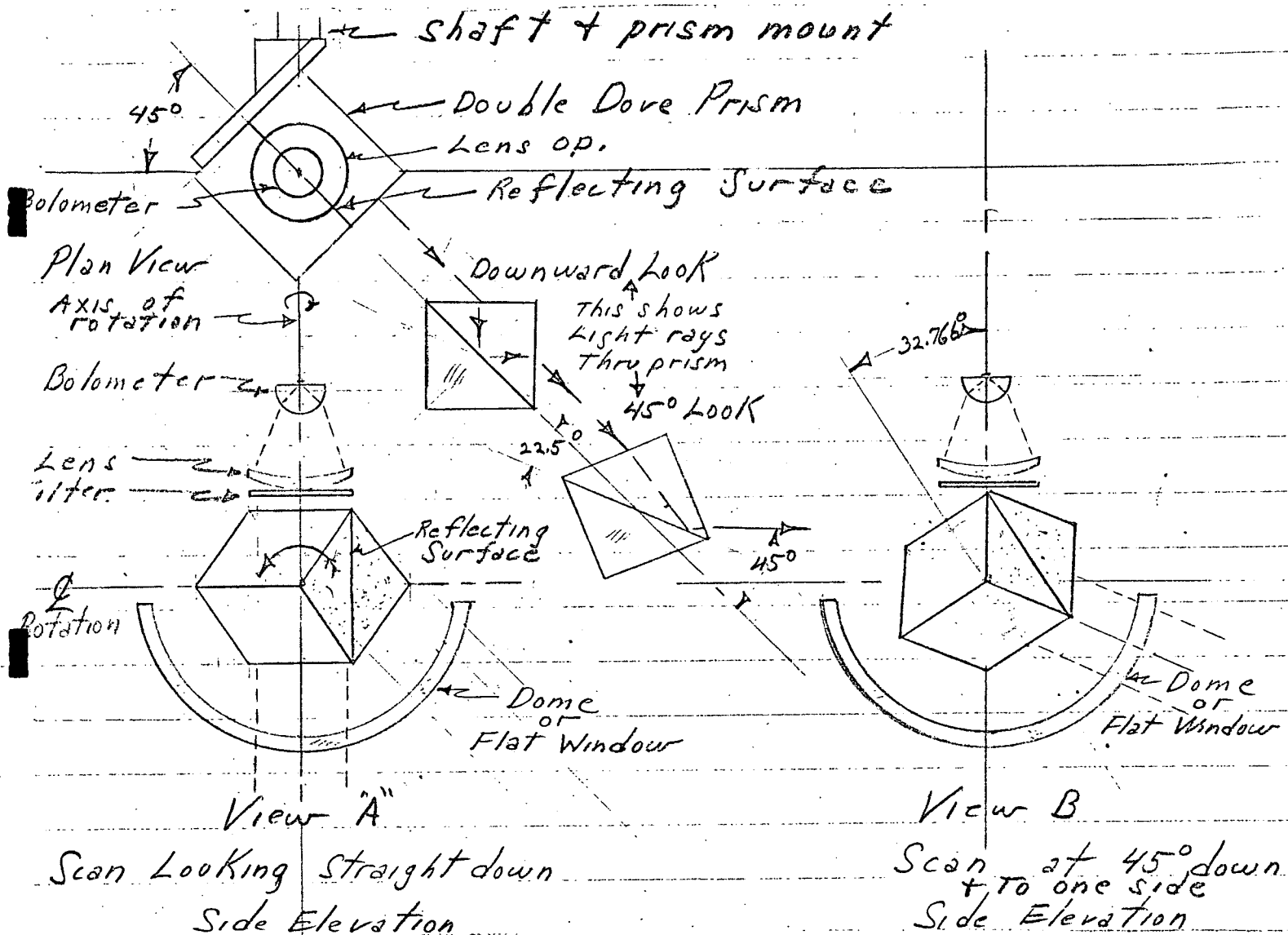
FIGURE II

The optical scanner consists of a bolometer (detector), a lens, a filter and substrate (cut on), a rotating double dove prism and a dome (or window).

Figure II view "A" shows the scanner looking at the center of the earth (the intersection of the two scans).

Invented by
 Kennard Watson Harper   April 1, 1971

Witnessed and Understood by

1) *Pauli Jan* 4-8-71

2) *[signature]* 4-8-71

Figure II view "B" shows the scanner in a low orbit (at about 500 N. Mi. above the earth) looking at the horizon of the earth, about 45° from the vertical.

Figure II the view at 45° from the plan view shows the profile of the double dove prism and the 45° look shows it swung 22.5° to look at the edge of the earth.

Figure III shows the trace of rays through a typical double dove prism.

Scanner Aperture

Aperture to object (Same as Scanner)
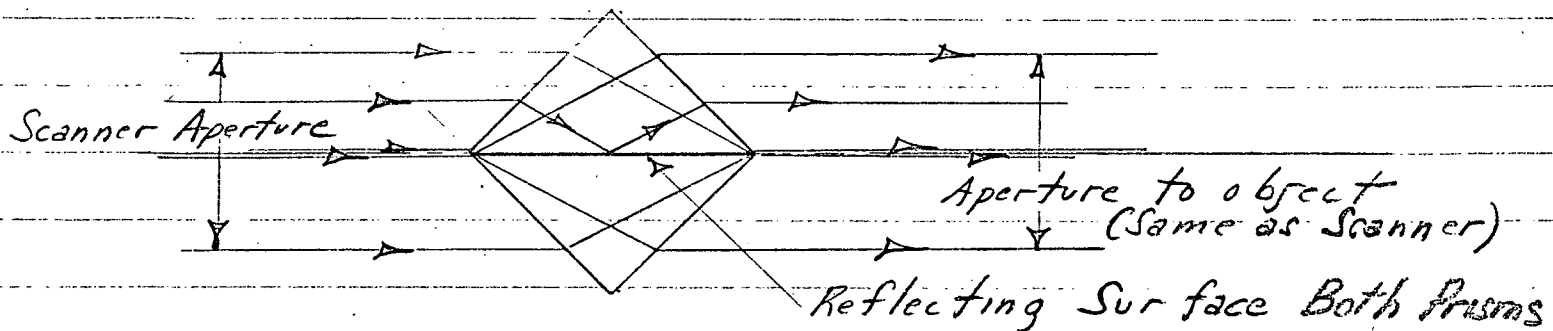
Reflecting Sur face Both Prisms

FIGURE 3

The index of refraction controls the prism length ratio to its height. The higher the index the shorter the prism. The prism design does have to be proportioned for its particular application. The face angles can be varied to accomplish this purpose.

Claims -

1) One scanner head has two scan paths to control both pitch and roll for any altitude of orbit.

2) One scanner drive with one axis of rotation can be oriented to its best position.

3) There is one complete scan (two axis) with one revolution of the scanner drive, which means longer bearing life.

4) The use of a double dove prism placed at a fixed angle of θ/2 Figure I controls angles of the scan pattern.

Invented by
 Kennard Watson Harper   April 1, 1971
Witnessed and Understood by

1)

2)

β

<u>Claims</u> - 5) The scan pattern angle θ Figure I can be varied by simply changing the swing of the reflecting hypotense from the basic 45° to the desired angle.

Invented by

Kennard Watson Harper   April 1, 1971

Witnessed and Understood by

1)

2)

PATENT DISCLOSURE

HORIZON SCANNER - ANY ALTITUDE SATELLITE

This is in addition to the disclosure of April 1, 1971.

This additional disclosure covers the special detector flake arrangement which is dictated by the tilting of the image due to the Double Dove Prism being tilted 45° to 50°.

When the dual flake (Sun rejection) bolometer is required the present two flake design will not work because the Double Dove Prism tilts the image 90°. The dual flakes will not cross the horizon simultaneously for both scans. That is, that two flakes are tangential to the earth.

This means that the arrangement of the flakes will be as shown by Figure 4 so that one pair will be tangential to the horizon on one leg of the crossing and the other pair will be tangential to the horizon on the other leg of the crossing. The middle flake ("C") is the common flake.

As the lens system flops the detector image over and around, Figure 5 demonstrates how this works.

The lens both inverts and reverts the individual flakes and whole pattern. This is shown by the intermediate aerial image.

In addition - the Double Dove Prism inverts the flakes and image due to the reflecting surface in the middle of the prism. This is shown by the last aerial image.

As the prism in its working position is tilted 45° to 50° from its normal position the flakes are tilted 90° to 100°. Figure 6 shows the Double Dove Prism tilted 45° which tilts the flakes 90°. If the prism were tilted 50° the flakes would be tilted 100°.

By placing these flakes as shown the first two flakes such as the Number 2 and common flake intersect the earth, and the Number 1 flake (trailing) can be blanked out as it isn't important. On the other half of the scan the Number 1 and common flakes will intersect the earth and the Number 2 (trailing) flake can be blanked out. The intersecting pair of flakes in each case is very nearly tangential to the earth. The tangential intersection is better at the higher altitude than at the lower altitudes.

Invented by
 Kennard Watson Harper   June 2, 1971 /Kennard Watson Harper
                                        6-2-71
Witnessed and Understood by
 1) Paul Brown   6-2-71
 2)   6/21-71

Claim - That the three flake bolometer arranged as shown is required for sun rejection in the Double Dove Prism disclosure.

The three flakes can be rotated into other axes with the same result except it is felt that the logical operation dictates that the blanked or unused flake be the trailing one.

Invented by
 Kennard Watson Harper   June 2, 1971   *Kennard Watson Harper*
                                         6-2-71
Witnessed and Understood by
1) *David Brennan*   6-2-71
2) *[signature]*   6/2/71
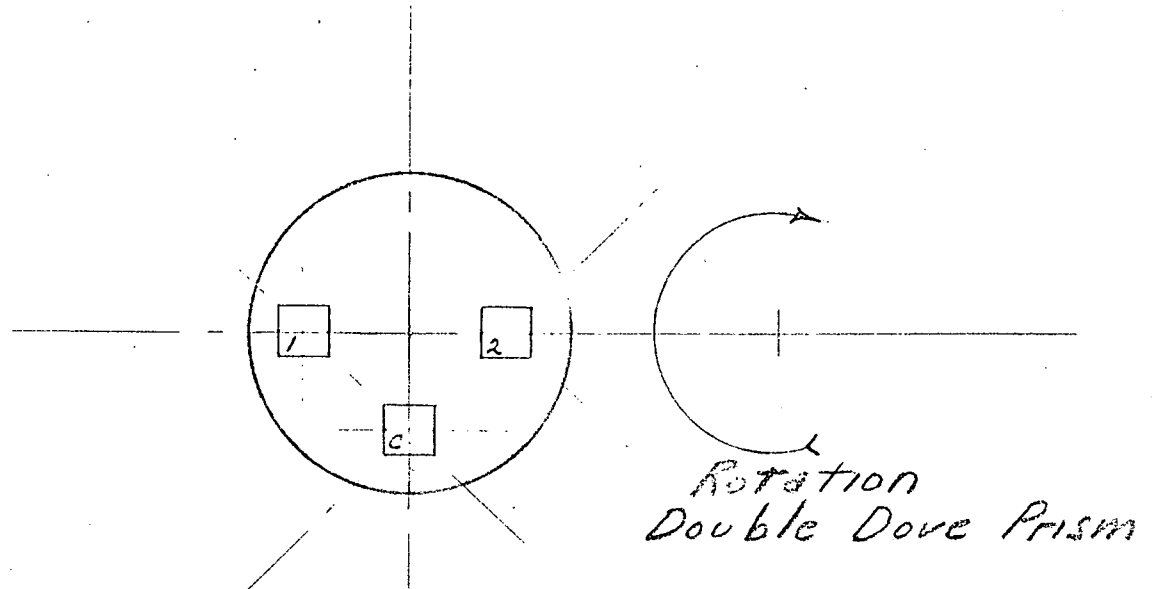
Rotation
Double Dove Prism

Figure No. 4

Bolometer positioned in scanner
Looking at earth

Three flake bolometer
"C" flake is common to both
#1 + #2 flakes

Invented by
Kennard Watson Harper Apr. 26, 1971 Kennard Watson Harper
                                                              6-2-71
Witnessed + understood by

David Son_____ 6-2-71
E.G. Mitchell   6-2-71

#2 Bolometer
#1
"c"
Lens

"c"
Inversion + Reversion
#1 of Flakes
after Lens
#2

Double Dove Prism

#1
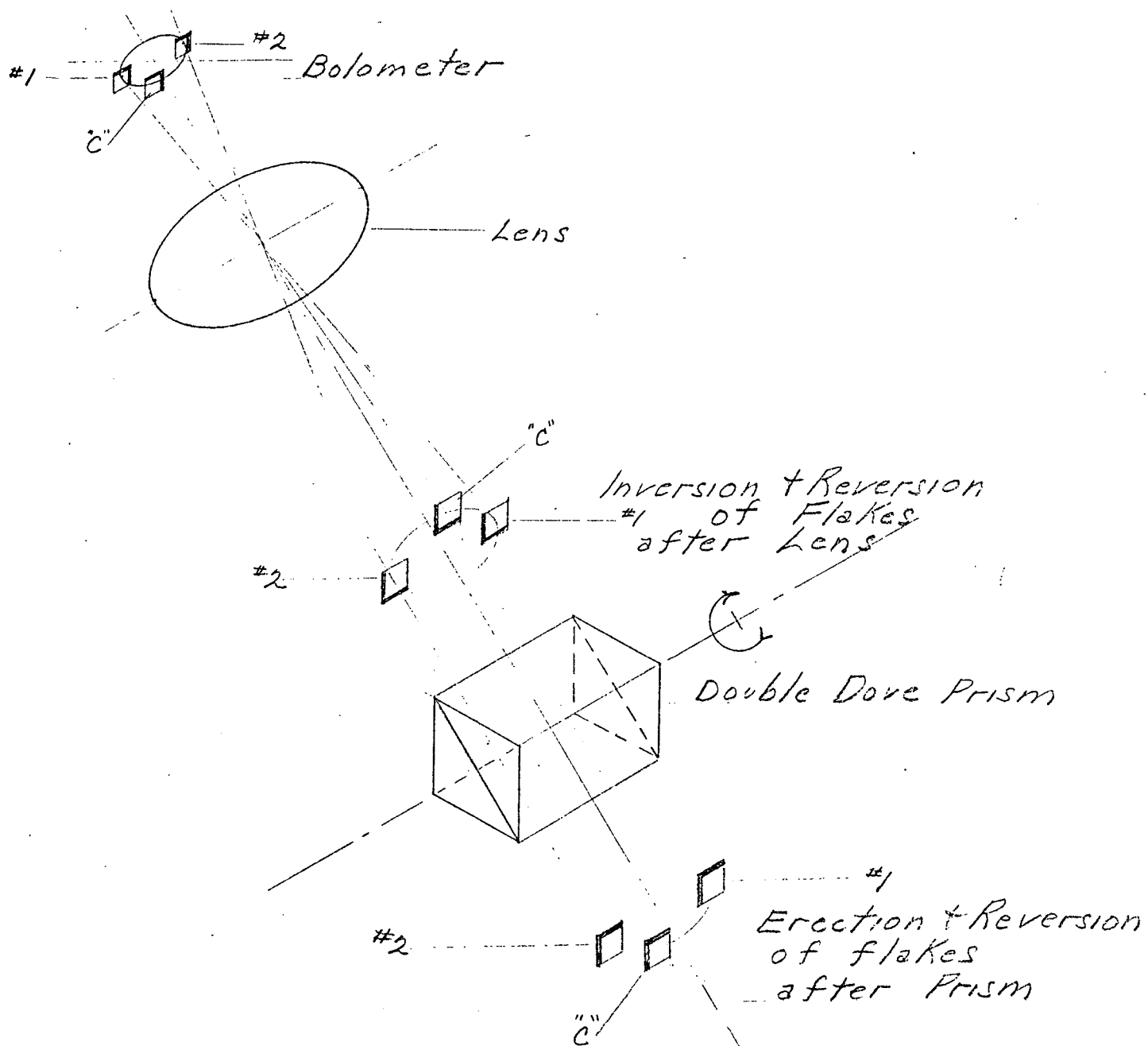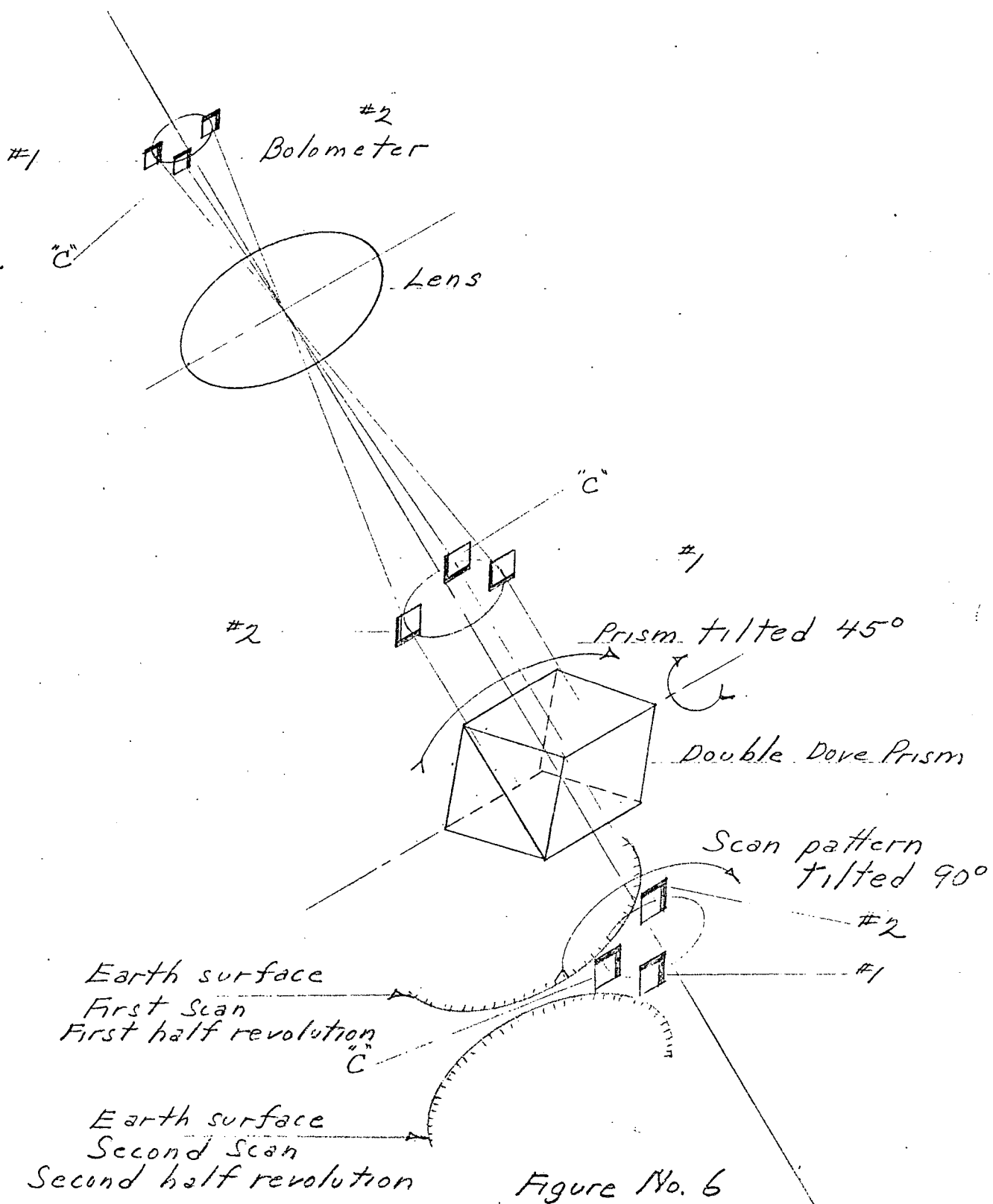Erection + Reversion
#2 of flakes
after Prism
"c"

Figure No. 5

Invented by
Kennard Watson Harper   Apr. 26, 1971 / Kennard Watson Harper
Witnessed + under-stood by   6-2-71
David Bonner   6-2-71
P.C. Mitchell   6-2-71

#1

#2
Bolometer

"C"

Lens

"C"

#1

#2

Prism tilted 45°

Double Dove Prism

Scan pattern
tilted 90°

#2

#1

Earth surface
First Scan
First half revolution

"C"

Earth surface
Second Scan
Second half revolution

Figure No. 6

Invented by
Kennard Watson Harper Apr. 26, 1971 / Kennard Watson Harper
Witnessed & understood by   6-2-71

David _____ 6-2-71

_____ 6-2-71

April 13, 1971

MEMO

TO: File

FROM: D. Sonnabend

SUBJECT: X Scanner Beam Analysis

Consider a horizon scanner of the type described by Harper.* The purpose of this memo is to determine the direction of the beam relative to the spacecraft at any position of the shaft or wheel. Assume a coordinate system with base vectors E_β^B fixed to the case. Suppose that the direction of the telescope as seen from the prism is E_1^B. Also suppose that the shaft axis on which the prism rotates is E_3^B.

Now, for the purpose of analysis, the double dove prism acts like a mirror whose reflecting surface is the total internal reflecting surface of the prisms. Both sides of this "mirror" reflect. We suppose that the mirror is fixed in a rotating coordinate system with base vectors E_α^W which must obey the relation:

$$E_\alpha^W = R_{\alpha\beta}^3 (\eta) E_\beta^B \qquad (1)$$

Within E^W, the mirror is completely described by its unit normal U, which for definiteness we take as:

$$U = E_2^W c\lambda + E_3^W s\lambda = R_{2\alpha}^1 (\lambda) E_\alpha^W \qquad (2)$$

This definition is completely general, as any other arrangement at the same cant λ differs only by some fixed phase angle added to η. The mirror cant λ is suggested as $\pi/4$ by Harper, but it is not necessary to pin it down in this analysis.

*Patent Disclosure 4-6-71, by Kennard W. Harper

We define the beam direction by the unit vector V, and note that it is determined by reflecting the telescope E_1^B off the "mirror". Thus V must obey the vector relation:

$$E_1^B \times U = U \times V \tag{3}$$

Observe that replacing U by -U leads to the same result, so that both scans are included in the solution. It is easiest to express each vector in E^B. Accordingly:

$$U_\alpha^B = R_{\mu\alpha}^3(\eta)U_\mu^W = R_{2\mu}^1(\lambda)R_{\mu\alpha}^3(\eta)$$

$$U^B = [-c\lambda s\eta, \ c\lambda c\eta, \ s\lambda] \tag{4}$$

The general cross product relation for $C = A \times B$ is

$$C_\alpha = \varepsilon_{\alpha\beta\gamma}A_\beta B\gamma; \quad \text{and in this case we have:}$$

$$\varepsilon_{\alpha\beta\gamma}U_\beta^B V_\gamma^B = \varepsilon_{\alpha\beta\gamma}\delta_\beta^1 U_\gamma^B = \varepsilon_{\alpha 1\beta} U_\beta^B$$

so that:

$$(\varepsilon_{\alpha\beta\gamma}V_\gamma^B + \varepsilon_{\alpha\beta 1})U_\beta^B = 0 \tag{5}$$

Now combining (4) and (5), a relation between the V_γ^B is produced for each value of α. Working this out:

$$V_2^B s\lambda = V_3^B c\lambda c\eta \tag{6}$$

$$(V_1^B + 1)s\lambda + V_3^B c\lambda s\eta = 0 \tag{7}$$

$$(V_1^B + 1)c\eta + V_2^B s\eta = 0 \tag{8}$$

The systems (6)-(8) is homogenous in the variables $V_1^B + 1$, V_2^B, and V_3^B; so it is solvable only if the determinant of coefficients vanishes, and then only for the ratios of the V_α^B. That the determinant vanishes is easily verified; so a further relation between the V_α^B will produce a solution.

The relation $E_1^B \cdot U = U \cdot V$ is true, but is is coplanar to (6)-(8), so it is no help.

The way out is that no use has been made of $V \cdot V = 1$. Substituting (6) and (7) into this gives:

$$(V_3^B c\lambda s\eta + s\lambda)^2 + (V_3^B c\lambda c\eta)^2 + (V_3^B s\lambda)^2 = s^2\lambda \qquad (9)$$

The solution $V_3^B = V_2^B = 0$, $V_1^B = -1$ is readily seen to be irrelevant. The remaining solution works out to:

$$V_1^B = 2c^2\lambda s^2\eta - 1 \qquad (10)$$

$$V_2^B = -2c^2\lambda s\eta c\eta \qquad (11)$$

$$V_3^B = -2s\lambda c\lambda s\eta \qquad (12)$$

Equations (10)-(12) are a complete description of the beam for an X type horizon scanner.

It is of interest to determine the directions of motion of the beams. To find these we find the change in V^B due to a change $d\eta$:

$$dV^B = 2c\lambda d\eta [c\lambda s(2\eta), -c\lambda c(2\eta), -s\lambda c\eta] \qquad (13)$$

These components are the direction numbers of the instantaneous rotation axis of the beam. Now at either $\eta = 0$ or $\eta = \pi$ we have $V^B = [-1,0,0]$; i.e., the beam is straight down from the telescope. At these values:

$$dV^B(0) = 2c\lambda d\eta [0, -c\lambda, -s\lambda]$$

$$dV^B(\pi) = 2c\lambda d\eta [0, -c\lambda, s\lambda]$$

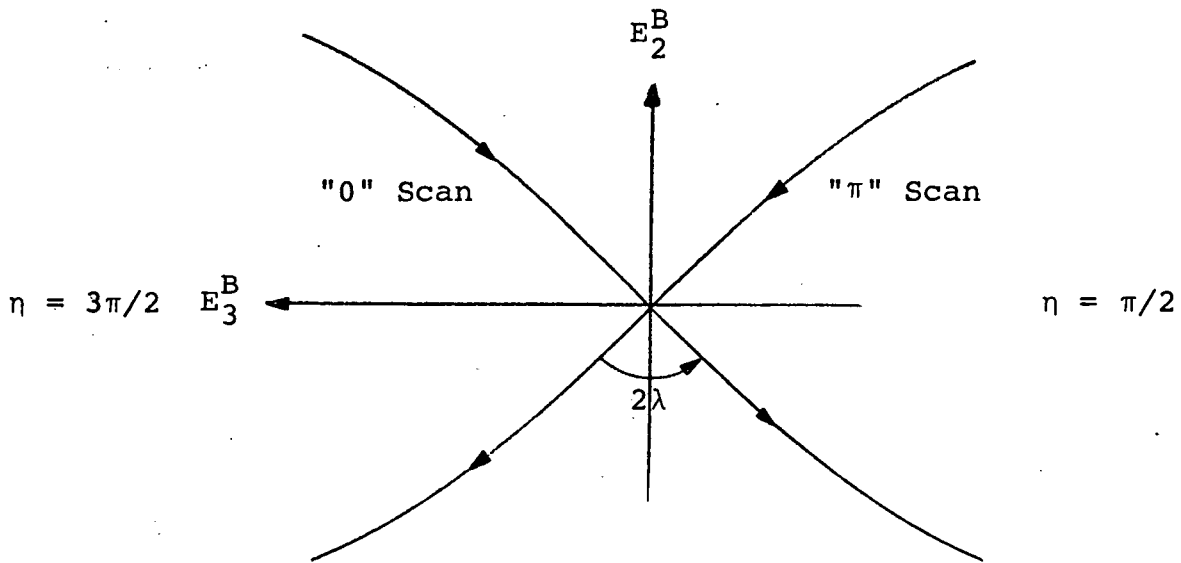Since the bracketed quantities are each of unit length, the angle x between the scans where they cross is:

$$cx = c^2\lambda - s^2\lambda = c(2\lambda) \qquad (14)$$

Thus the beam crosses itself at an angle 2λ.

As a final note, if $\lambda = \pi/4$, the beam crosses itself at right angles and V^B has the special form:

$$V^B = -\left[c^2\eta, \; sncn, \; sn \right] \qquad (15)$$

Looking down from the telescope the projection of the beam will look essentially like this:

MEMO

TO: File

FROM: D. Sonnabend

SUBJECT: X Scanner Computational Analysis for Small Angles

In a previous memo* the Harper X Scanner was analyzed to determine the direction of the beam relative to its outer case, as a function of wheel (prism) position. Here this analysis will be extended to find the conditions at horizon crossings, and use these to obtain computational equations for the determination of satellite attitude and altitude. The main results are valid only for small attitude errors.

The notations of the previous memo will be preserved intact, but augmented to describe satellite attitude. We assume a reference or trajectory coordinate system with unit base vectors E_α^T defined so that E_1^T is along the outward pointing radius vector from the earth's center, E_3^T is along the orbit normal as though the orbit were a right hand screw, and E_2^T is forward in the orbit plane to complete a right handed system. We suppose that the body fixed system E^B used in the previous memo is related to E^T by:

$$E_\alpha^B = F_{\alpha\beta} E_\beta^T \tag{1}$$

and for simplicity, suppose that $F = I$ corresponds to no attitude error. This implies that the telescope looks straight down $(-E_1^T)$ when there is no error.

Since a horizon scanner can give no information about yaw (i.e., rotation about E_1^T) it will prove convenient to factor F as follows:

$$F = AR^1(\psi) \tag{2}$$

*"X Scanner Analysis", D. Sonnabend 4-13-71

To assure that the rotation A contains no yaw component, we will require that ψ be so chosen that the eigenvector of A is orthogonal to E_1^T whenever $F \neq I$; if $F = I$ then $A = I$ and $\psi = 0$.

The angle α between the beam axis V and the nadir $-E_1^T$ is:

$$c\gamma = -V \cdot E_1^T = -V_\alpha^B E_\alpha^B \cdot E_1^T$$

and substituting from (1) this works out as:

$$c\gamma = -V_\alpha^B F_{\alpha 1}$$

Then using (2) we have:

$$c\gamma = -V_\alpha^B A_{\alpha 1} \tag{3}$$

The point of using A rather than F is that the analysis is not cluttered with an angle which cannot be determined.

In practice, (3) is useful whenever the beam crosses a horizon so that γ has the special value γ_e given by:

$$s\gamma_e = (1 + h/r_e)^{-1} \tag{4}$$

where h is the satellite altitude and r_e is the radius of the earth. Various effects such as clouds and oblateness introduce errors through (4) in that the horizons may not be affected symmetrically; but these will not be considered in this memo.

From the previous memo we may substitute the expressions for the V^B in terms of the prism angle λ and the wheel position η. When this is done each horizon crossing yields this relation:

$$(1 - 2c^2\lambda s^2\eta)A_{11} + 2c^2\lambda s\eta c\eta A_{21} + 2s\lambda c\lambda s\eta A_{31} = c\gamma_e \tag{5}$$

If the attitude error is not too great, there will be four distinct values of η corresponding to the four separate horizon crossings on each turn of the wheel. Thus four, not necessarily independent, relations exist for computing attitude. In sorting these out it is convenient to define the angle η_o by the relation:

$$c\gamma_e = 1 - 2c^2\lambda s^2\eta_o \tag{6}$$

Only the first quadrant value of η_o will be used $(0 < \eta_o \leq \pi/2)$. Now for the case of no error, $A = I$ and it follows that $s^2\eta = s^2\eta_o$. The four possible solutions obviously correspond to the four horizon crossings, and one will occur in each quadrant. Thus, for no error, we can label the four η values by quadrant:

$$\eta_1 = \eta_o; \quad \eta_2 = \pi - \eta_o; \quad \eta_3 = -\pi + \eta_o; \quad \eta_4 = -\eta_o$$

The physical significance of η_o is now evident -- it is the rotation of the wheel from when the beam is vertical to when it crosses the horizon, if there is no attitude error. It depends only on satellite altitude.

Using this null case as a guide, we can in general define a set of variations $\delta\eta_i$ from null by these relations:

$$\eta_1 = \eta_o + \delta\eta_1; \quad \eta_2 = \pi - \eta_o - \delta\eta_2; \quad \eta_3 = -\pi + \eta_o + \delta\eta_3;$$

$$\eta_4 = -\eta_o - \delta\eta_4 \tag{7}$$

In general, the $\delta\eta_i$ will be small if the rotation A is small. Returning now to A, for small rotations, any set of parameters will serve for computation, with equal facility. Accordingly, if we resolve A into pitch θ and roll ϕ by either $A = R^3(\theta)R^2(\phi)$ or $A = R^2(\phi)R^3(\theta)$, then for small angles we have to first order:

$$A_{11} = 1, \quad A_{21} = -\theta, \quad A_{31} = \phi$$

Thus, from (5) and (6):

$$\phi \tan \lambda s \eta - \theta s \eta c \eta = s^2 \eta - s^2 \eta_o = s(\eta + \eta_o) s(\eta - \eta_o) \qquad (8)$$

It is clear that one of the terms on the right of (8) must be small. Substituting in the four definitions (7) clears this up, and after rejecting all second order terms the four equations become:

$$\phi \tan \lambda - \theta c \eta_o = 2 \delta \eta_1 c \eta_o = 2(\eta_1 - \eta_o) c \eta_o \qquad (9)$$

$$\phi \tan \lambda + \theta c \eta_o = 2 \delta \eta_2 c \eta_o = 2(\pi - \eta_2 - \eta_o) c \eta_o \qquad (10)$$

$$\phi \tan \lambda + \theta c \eta_o = -2 \delta \eta_3 c \eta_o = 2(\eta_o - \eta_3 - \pi) c \eta_o \qquad (11)$$

$$\phi \tan \lambda - \theta c \eta_o = -2 \delta \eta_4 c \eta_o = 2(\eta_4 + \eta_o) c \eta_o \qquad (12)$$

The purpose of the $\delta \eta_i$ in linearizing these relations has been served, and as they are not directly observable, they will now be ignored. Viewing (9-12) as a system of four equations in the three unknowns ϕ, θ, and η_o, we see that there must be a relation between the observables η_i. To find this we compare (9) and (12) to get:

$$\eta_1 - \eta_o = \eta_4 + \eta_o$$

and also (10) and (11):

$$\pi - \eta_2 - \eta_o = \eta_o - \eta_3 - \pi$$

This shows that η_o can be calculated by any of these:

$$\eta_o = \frac{1}{2}(\eta_1 - \eta_4) = \pi + \frac{1}{2}(\eta_3 - \eta_2) = \frac{\pi}{2} + \frac{1}{4}(\eta_1 - \eta_2 + \eta_3 - \eta_4)$$

$$(13)$$

Thus the internal relation between the η_i is:

$$\eta_1 + \eta_2 - \eta_3 - \eta_4 = 2\pi \qquad (14)$$

In practice, the amount by which (14) fails may be looked upon as a measure of the lack of circularity of the observed horizon. It is also possible to use (14) to adjust the η_i to improve the system accuracy. If η_x is the actual value on the left of (14), then the adjusted values of the η_i are $\eta_i(2\pi/\eta_x)$.

The pitch angle θ can be found by subtracting (9) from (10) or (12) from (11):

$$\theta = \pi - \eta_1 - \eta_2 = -\pi - \eta_2 - \eta_4 = -\frac{1}{2}(\eta_1 + \eta_2 + \eta_3 \pm \eta_4) \tag{15}$$

Finally, the roll angle ϕ may be obtained by adding (9) to (11) or (10) to (12):

$$\phi = \cot\lambda c\eta_o(\eta_1 - \eta_3 - \pi) = \cot\lambda c\eta_o(\pi + \eta_4 - \eta_2)$$

$$= \frac{1}{2}\cot\lambda c\eta_o(\eta_1 - \eta_2 - \eta_3 + \eta_4) \tag{16}$$

In (16) the gain factor $\cot\lambda$ is fixed by the design of the scanner. The factor $c\eta_o$ however, is altitude dependent. It is possible to calculate it from (13), or, if the orbit is sufficiently circular, it may be precalculated and fixed, or its variation can be simply ignored. To see whether the latter may be feasible, this table, calculated from (4) and (6) may be useful. The radius γ_e is assumed to be 6367.37km = 3435.79nmi, and λ is taken to be $\pi/4$.

h-km	0	200	300	500	1000	5000	40,000
h-nmi	0	107.9	161.9	269.8	539.6	2698	21,584
γ_e	90°	75°49'	72°45'	68°0'	59°48'	34°4'	7°54'
$c\gamma_e$	0	.24491	.29659	.37459	.50303	.82839	.99052
η_o	90°	60°20'	57°0'	52°16'	44°50'	24°28'	5°35'
$c\eta_o$	0	.49488	.54460	.61203	.70925	.91016	.99525

Thus, above 300km, the gain will change by less than two to one for any higher altitude.